Using Topology to locate the position where fully Three-Dimensional Reconnection Occurs

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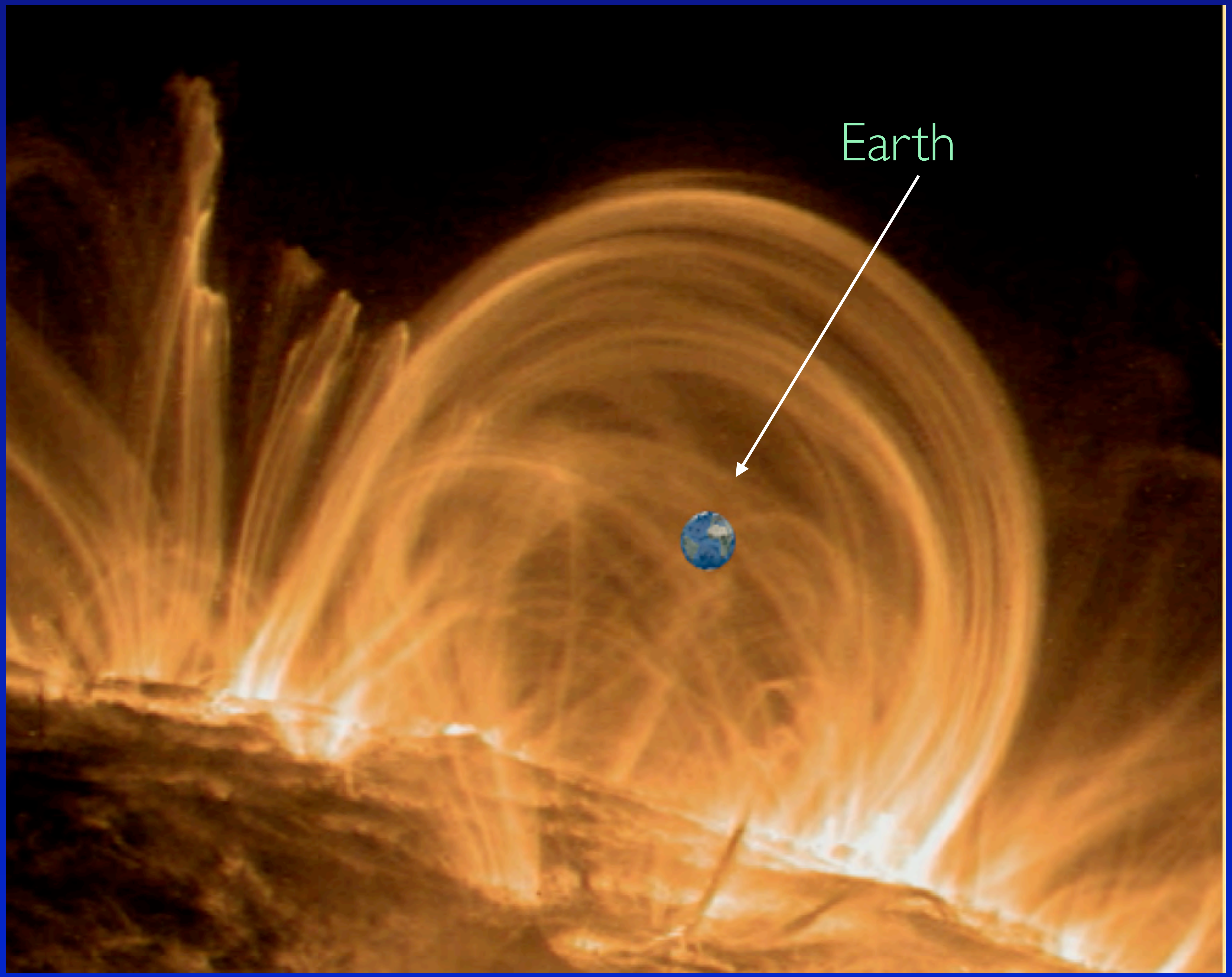
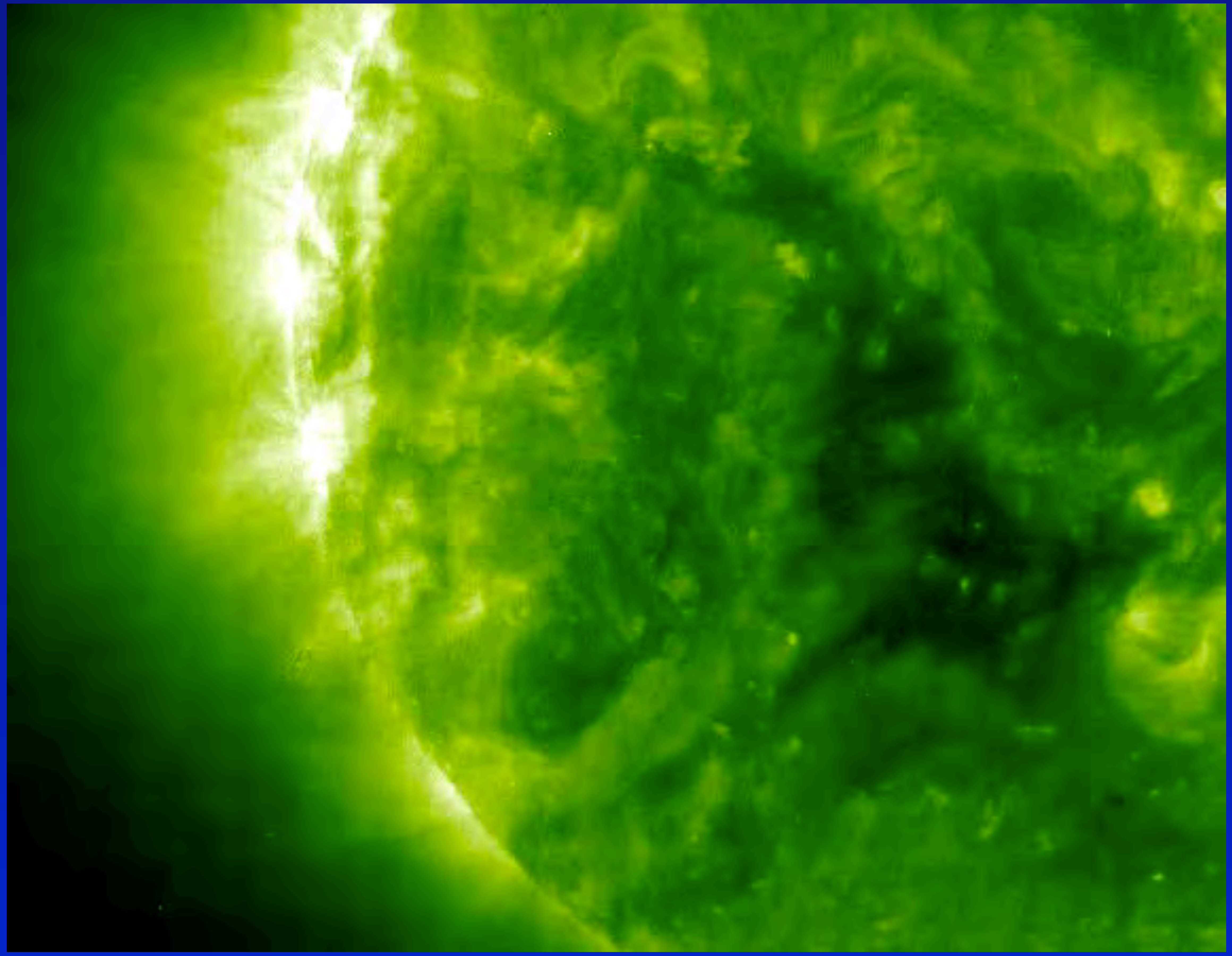
Springer Nature Physics 2:2187 (2020)

Magnetic Flux Ropes





Priest, Démoulin, "Three dimensional magnetic reconnection without null points. 1) Basic Theory of magnetic flipping" , JGR 100, (1995)

Priest, Hornig, Pontin, "On the nature of three-dimensional reconnection", JGR, 108,(2003)

Titov, Forbes, Priest, Mikiz, Linker, "Slip Squashing factors as a measure of three dimensional magnetic reconnection", Astrophys. Jour., 693 (2009)

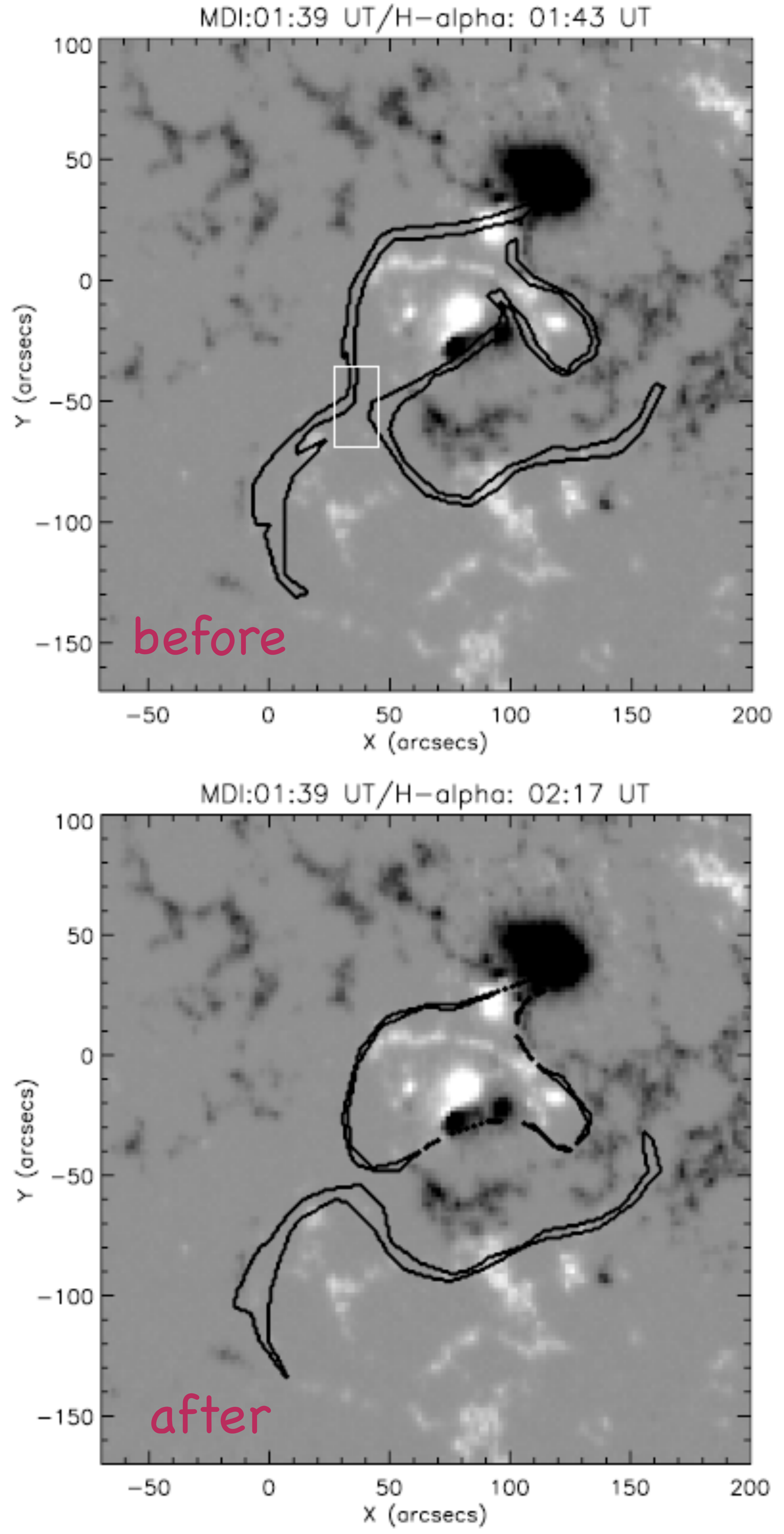
Démoulin, "Where will efficient energy release occur in3D magnetic reconnection?", Adv. Space Sci. 39 (2007)

Aulanier , "Coronal Heating and flaring in QSLs", Proceeds. IAU symposium 273 (2010)

Prior, Yeates, "Quantifying recollective activity in braided vector fields", Phys. Rev E, 98 (2018)

Data

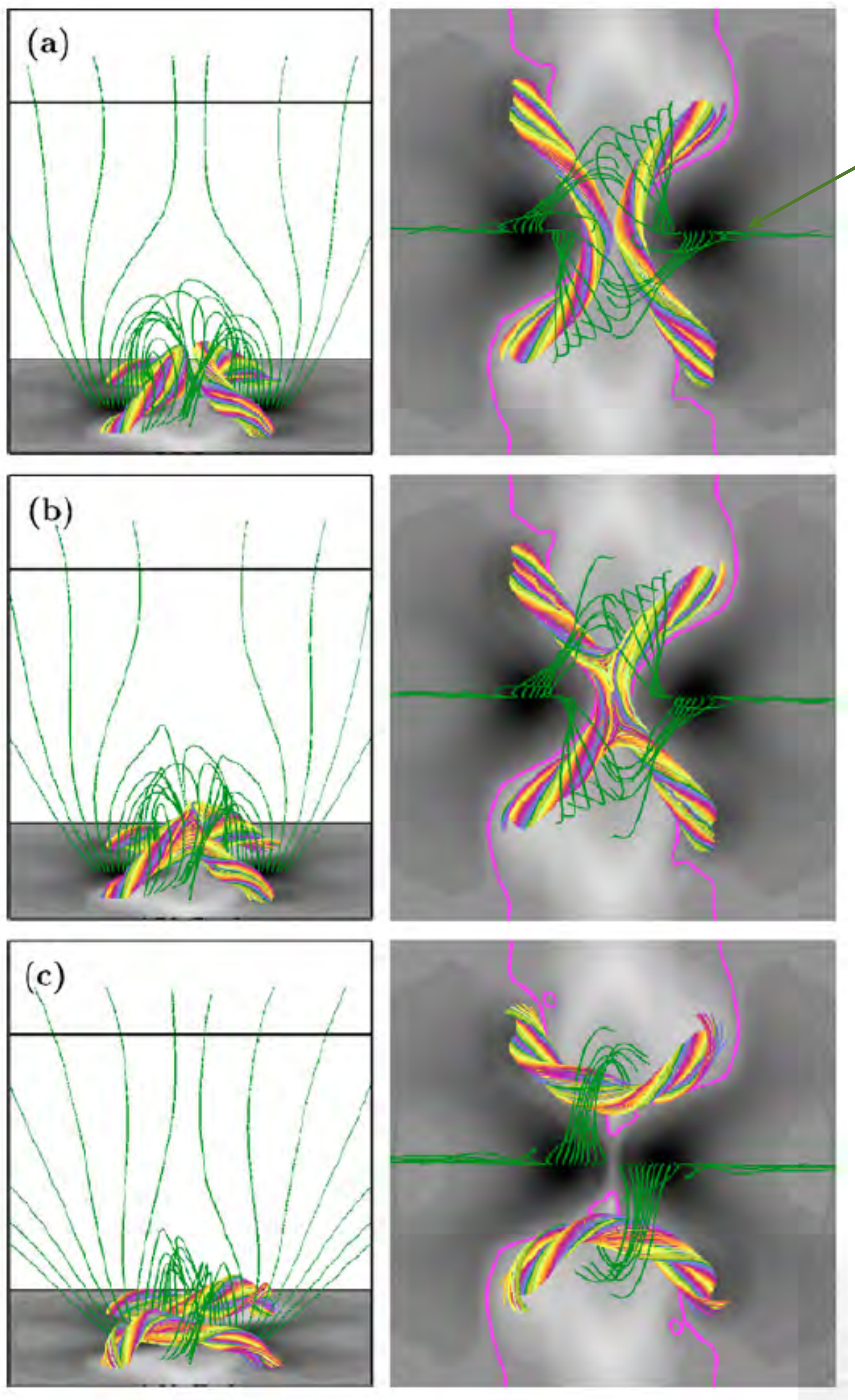
Magnetogram with filament shapes overlayed



Computer simulation

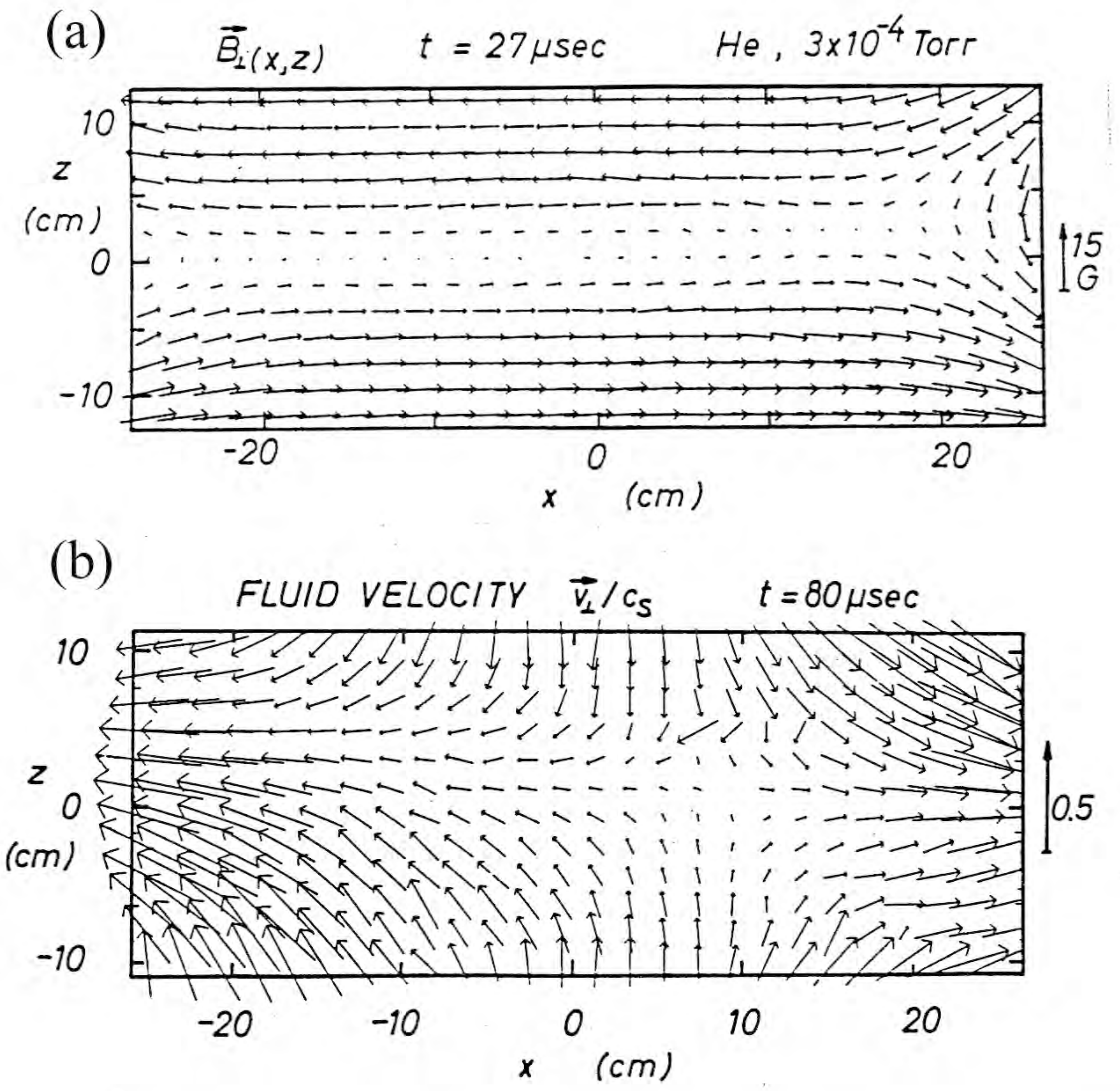
MHD $\beta = 0$

ambient field lines



Note in this case current in ropes is in opposite directions

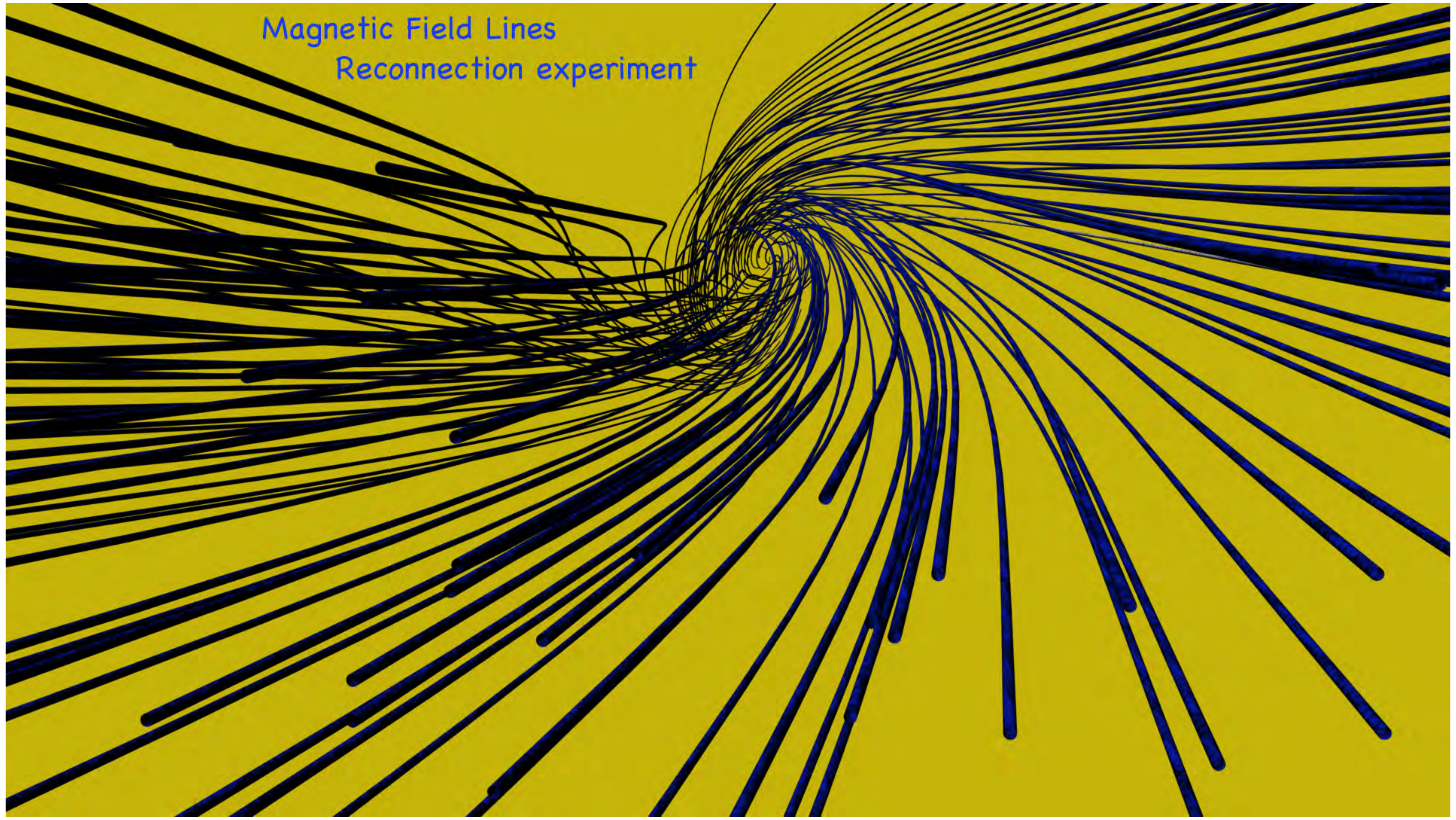
T. Torok et al, Filament Interaction modeled by flux rope reconnection, Ap. J. 728, 1, (2011)



2D reconnection

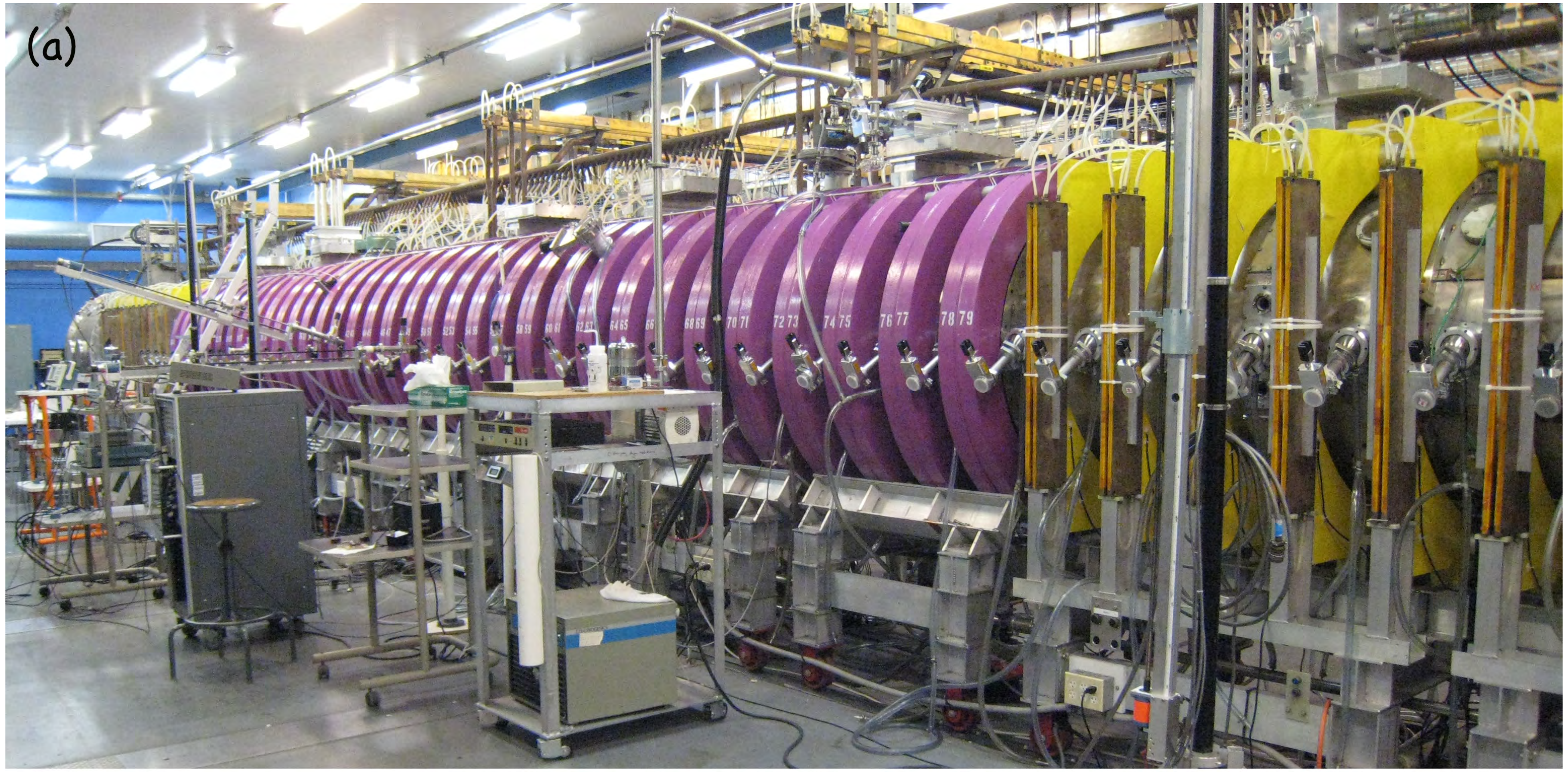
Gekelman, Stenzel 1980's
then MRX... others

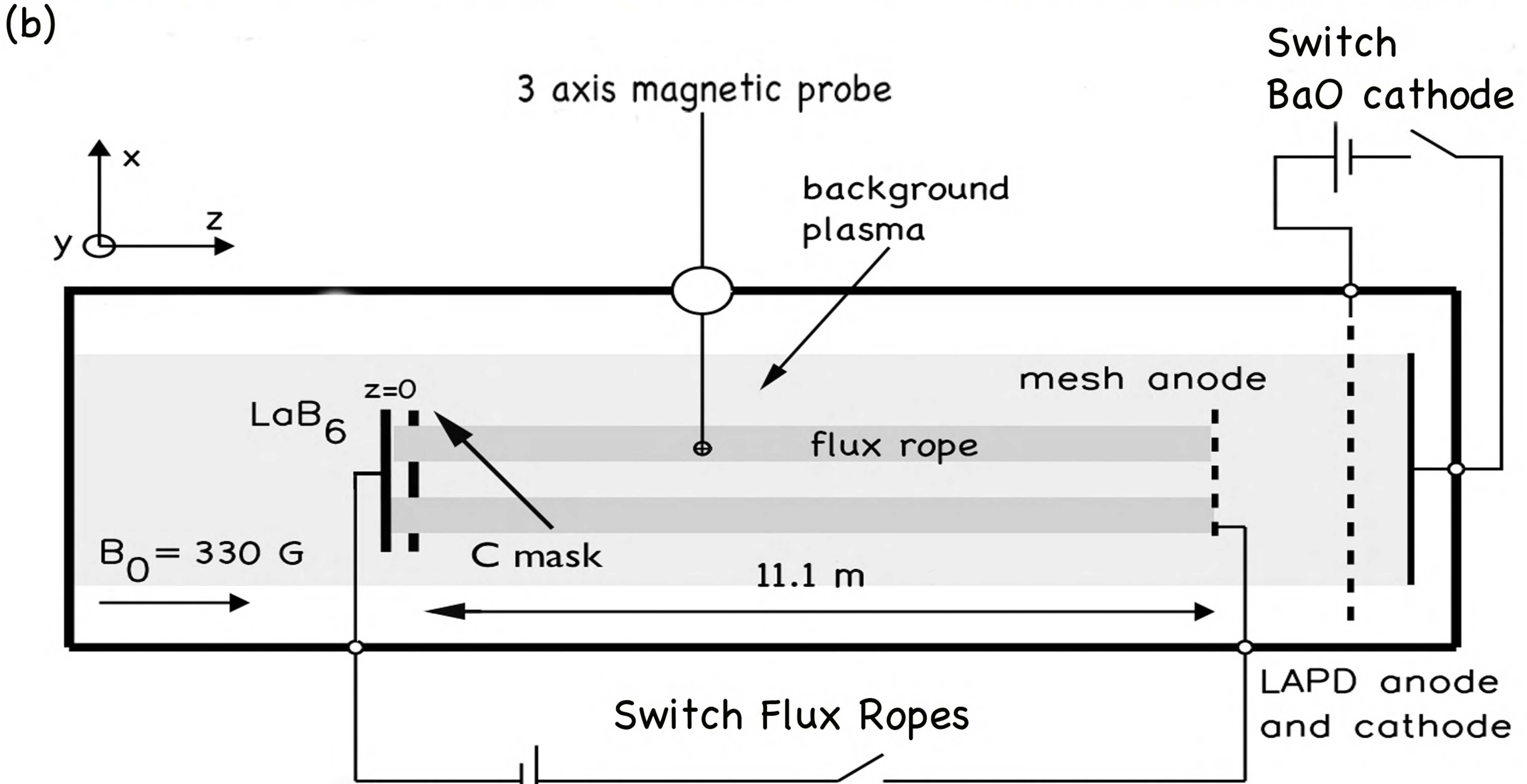
3D reconnection
large guide field
Bz/B⊥ ~ 10



No sheets or X points

where does reconnection occur?





Helium Plasma

$n = 2X10^{12}\ cm^{-3}$

T_e - 5 eV

T_I - 1 eV

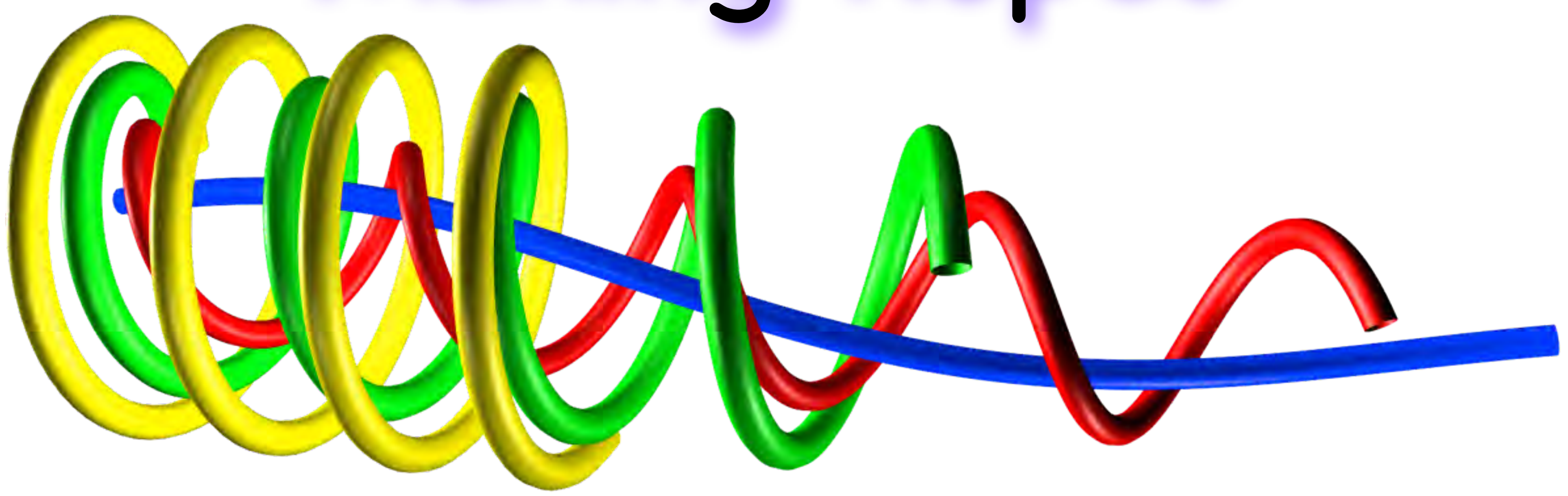
B_{0z} = 330 G

Measure:

$\vec{B}(\vec{r},t),\vec{v}_{flow}(\vec{r},t),n(\vec{r},t),T_e(\vec{r},t),\vec{E}(\vec{r},t)$

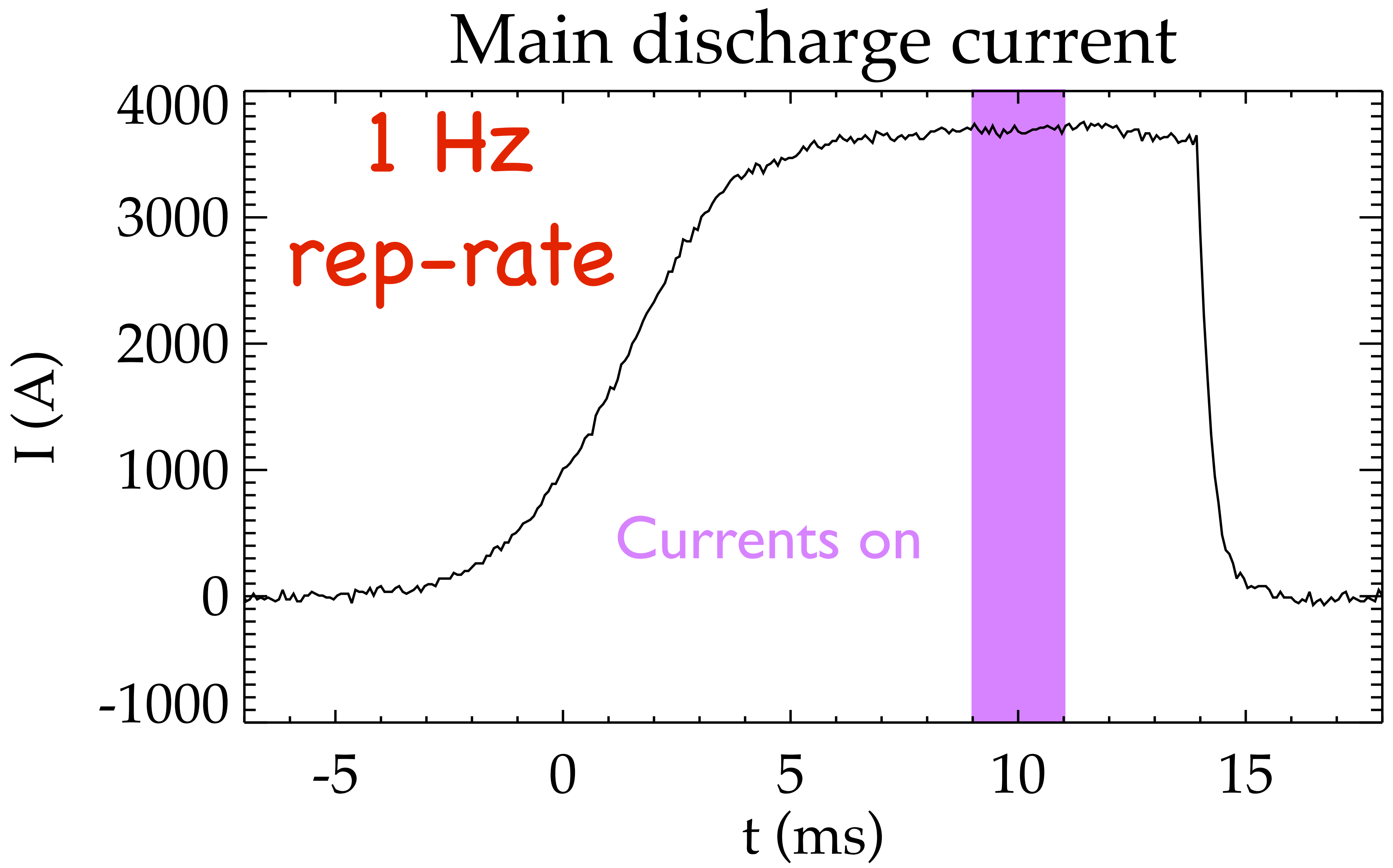
at 42,000 locations, 2.42 million shots

Making Ropes



The ropes are kink unstable

Discharge currents

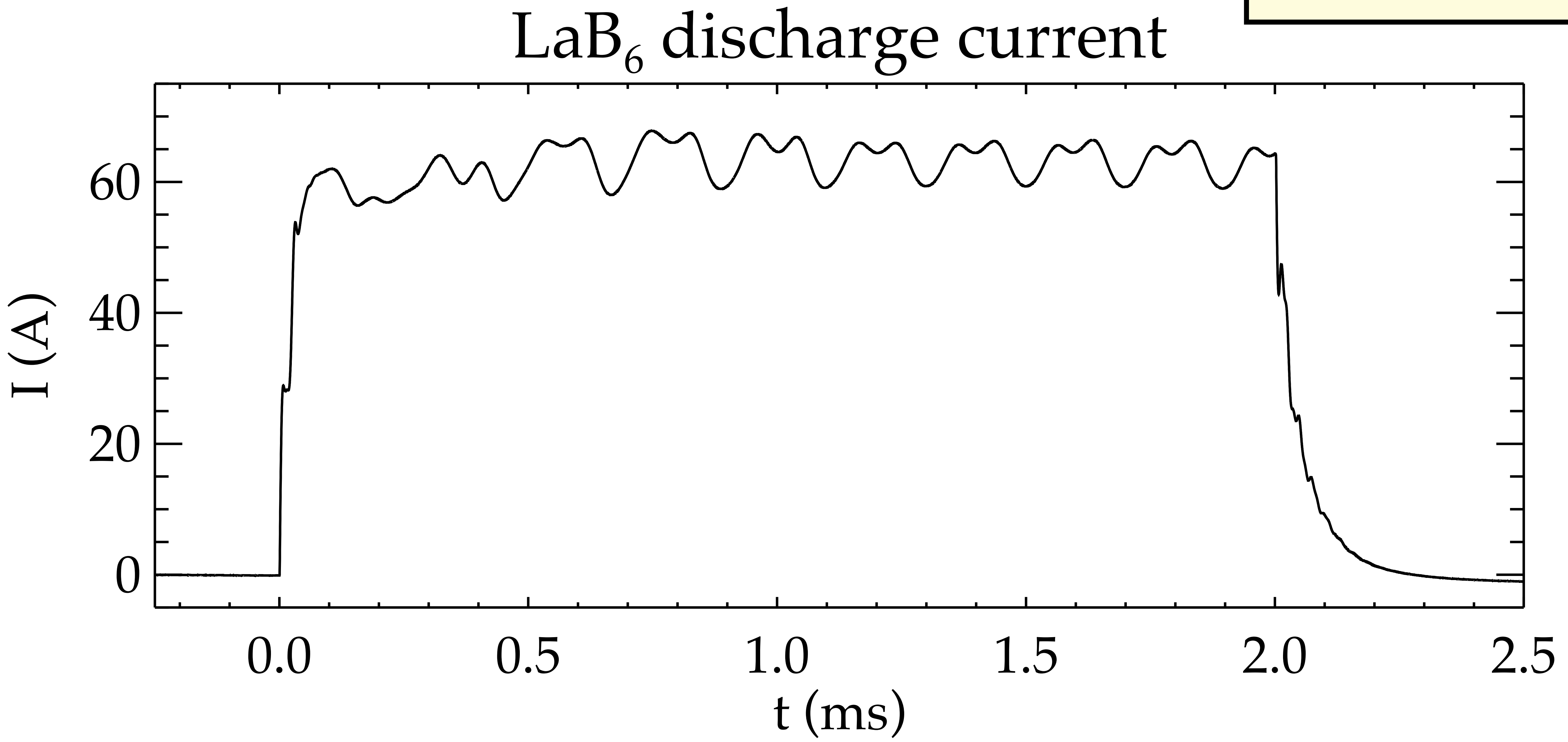


cathode biased to 120V for 2 ms during the main discharge. After 300 μs (~3τ_A), spontaneous oscillations are seen in the LaB_6 discharge current.

$$\gamma = \frac{c}{\omega_{pe}} = 3mm \qquad S = \frac{\mu_0 V_A L}{\eta} \cong 3\times10^3 - 1\times10^6$$

$$\gamma_I = \frac{c}{\omega_{pi}} = 28cm \qquad R_{ci} = 7.5mm$$

$$Q = \frac{2\pi a B_z}{L B_\theta} \cong 0.7 \quad B_z = 330G \quad B_\theta \simeq 6-10G$$



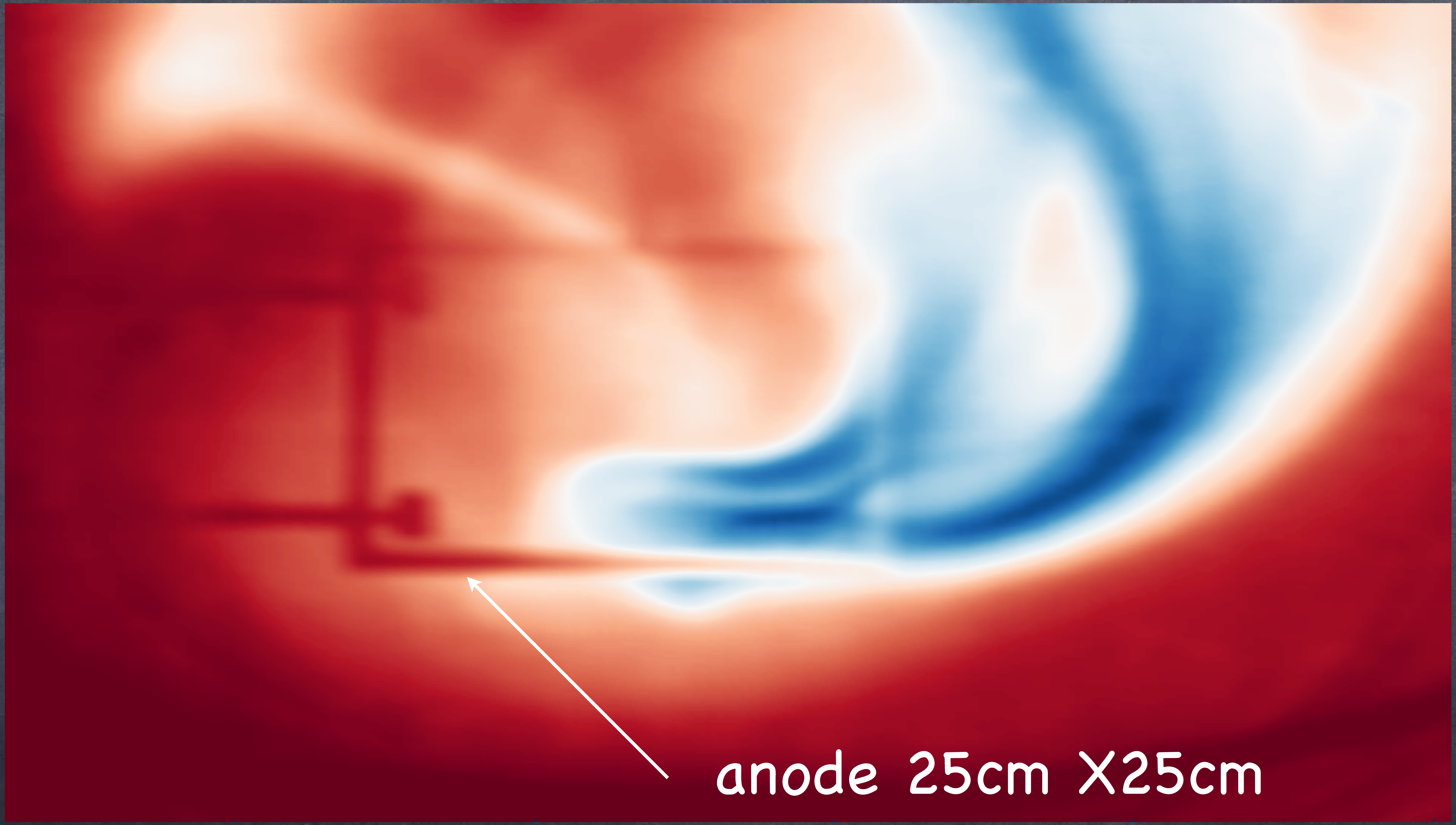
Q < 1

"kink" unstable

$$\frac{B_\theta}{aB_z} >> \frac{\omega}{V_A} \quad ; \quad \omega \approx \frac{2 v_z B_\theta}{a B_z}$$

Ruytov et al, PoP (2006)

Fast camera 1 ms exposure



Kink Dispersion relation

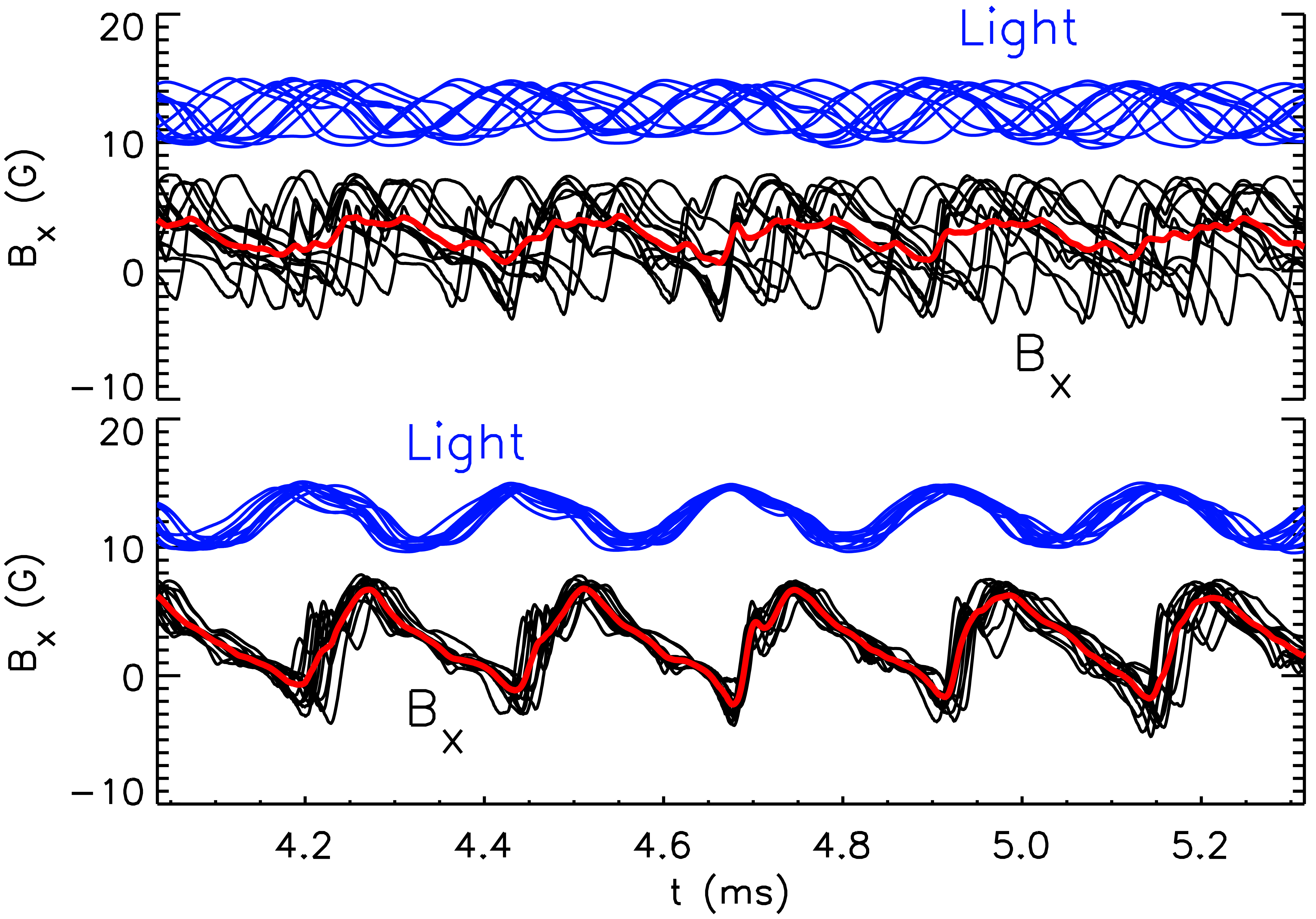
$$\tan\left(Lk_o\alpha\right) = -2ia$$

$$\alpha = \sqrt{\frac{1}{4} + \left(\frac{\omega}{\sqrt{2}k_0 V_A}\right)} \qquad \mathrm{k}_0 = \frac{B_\theta}{RB_{0z}}$$

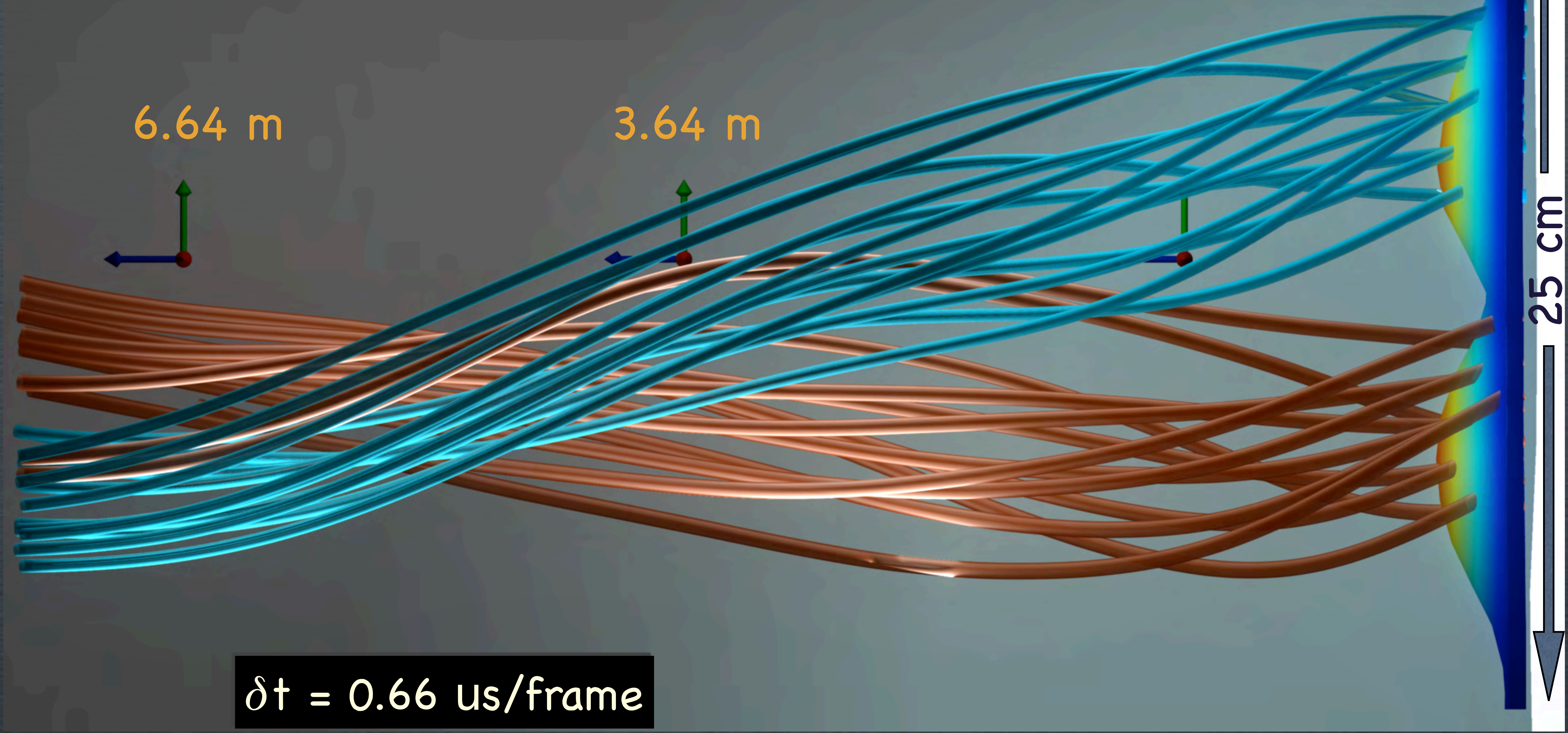
$4.7 < f < 7$ kHz

$f_{observed} = 5.2$ kHz

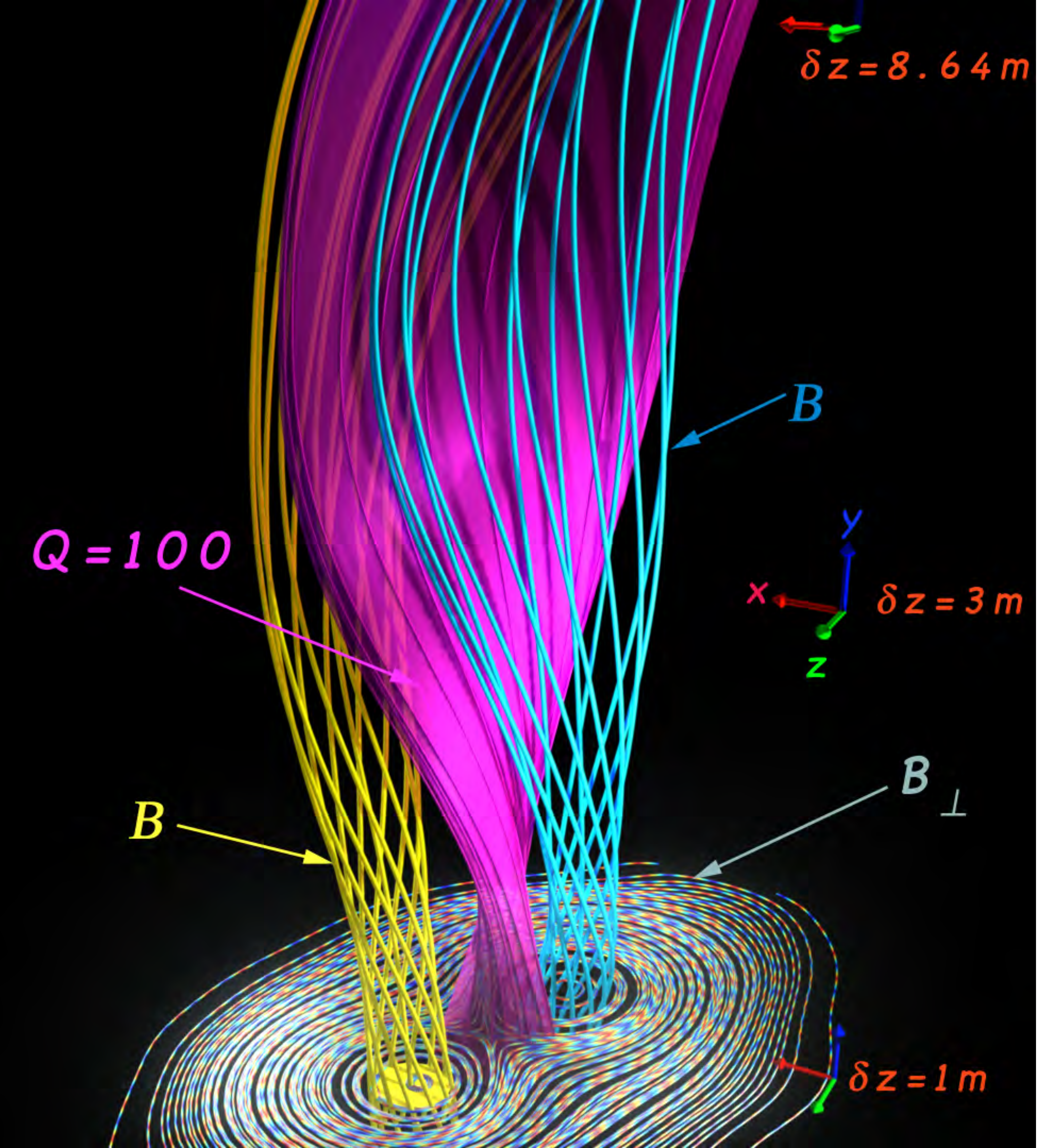
Phase Locking -"Conditional Trigger"



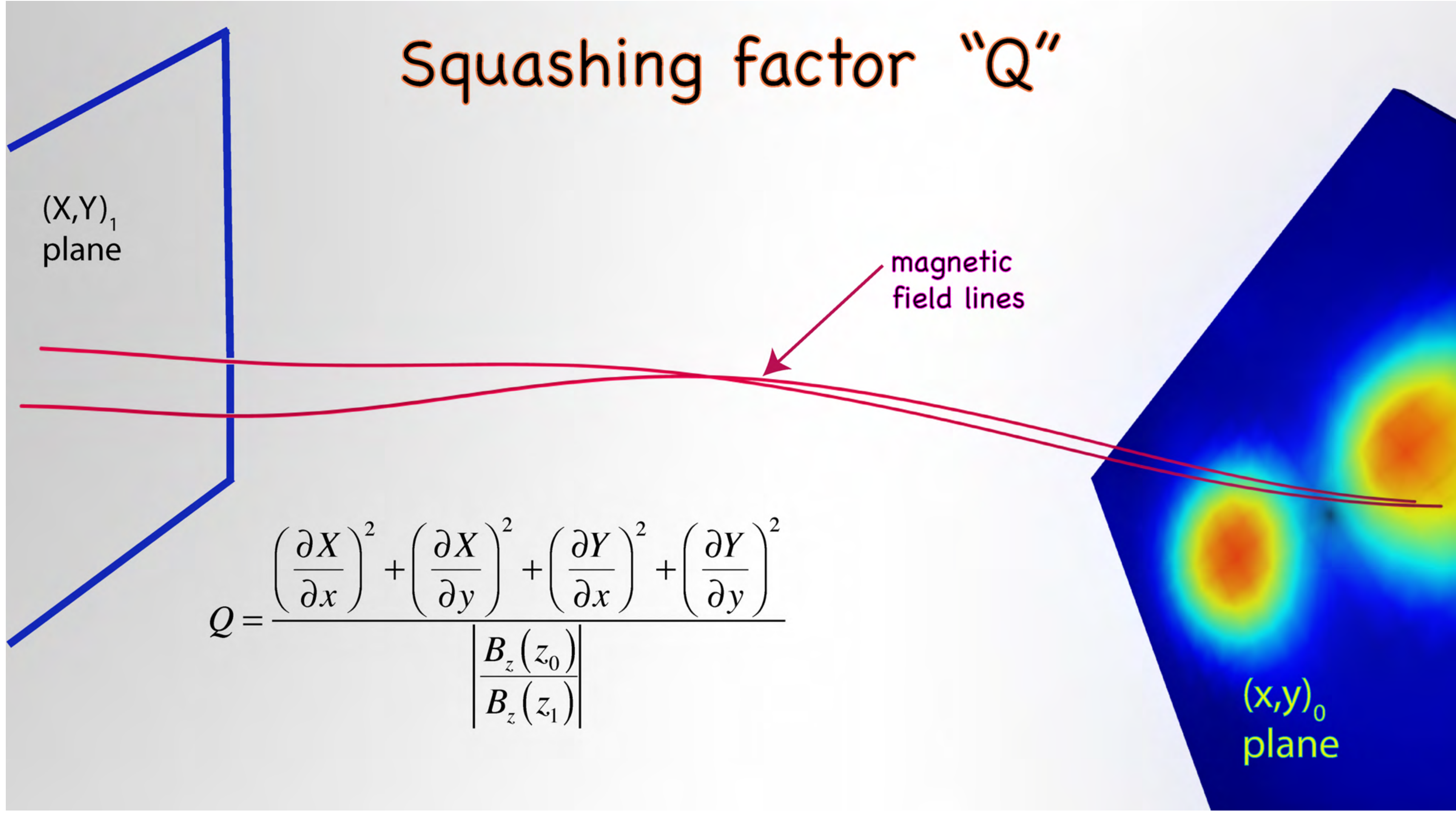
Magnetic Field Lines



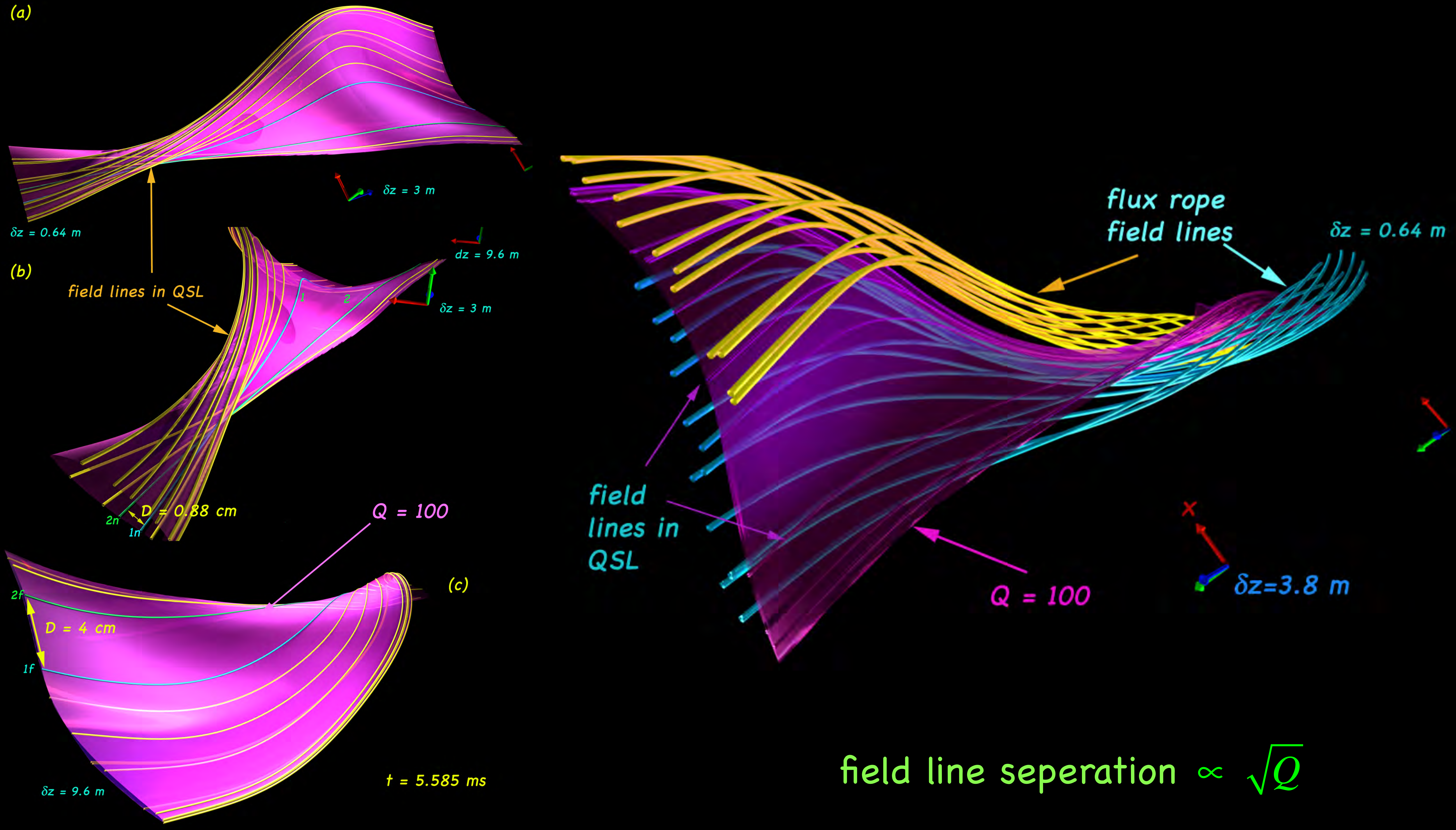
Rope Current
Rope Current



Q (squashing factor)
measures
the separation of initially
adjacent field lines
along the length of the
ropes



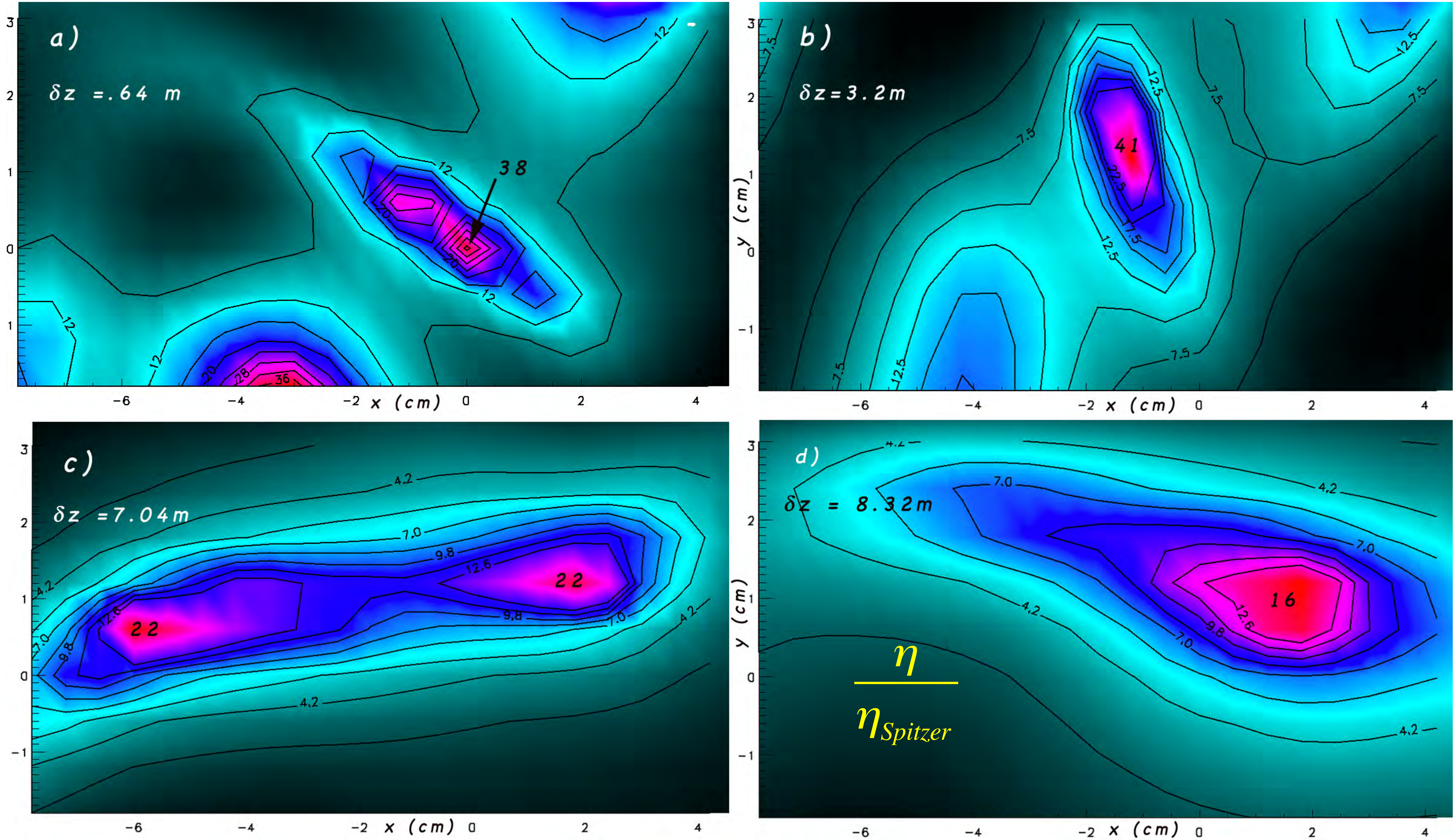
Reconnection occurs
somewhere in the QSL – where?


(a)
δz = 0.64 m
δz = 3 m
(b)
field lines in QSL
dz = 9.6 m
δz = 3 m
1
2
2n
1n
D = 0.88 cm
Q = 100
(c)
2f
D = 4 cm
1f
δz = 9.6 m
t = 5.585 ms
flux rope
field lines
δz = 0.64 m
field
lines in
QSL
Q = 100
x
δz=3.8 m
field line seperation ∝ √Q

δz=8.64m
B
Q=100
y
x
δz=3m
z
B ⊥
B
δz=1m

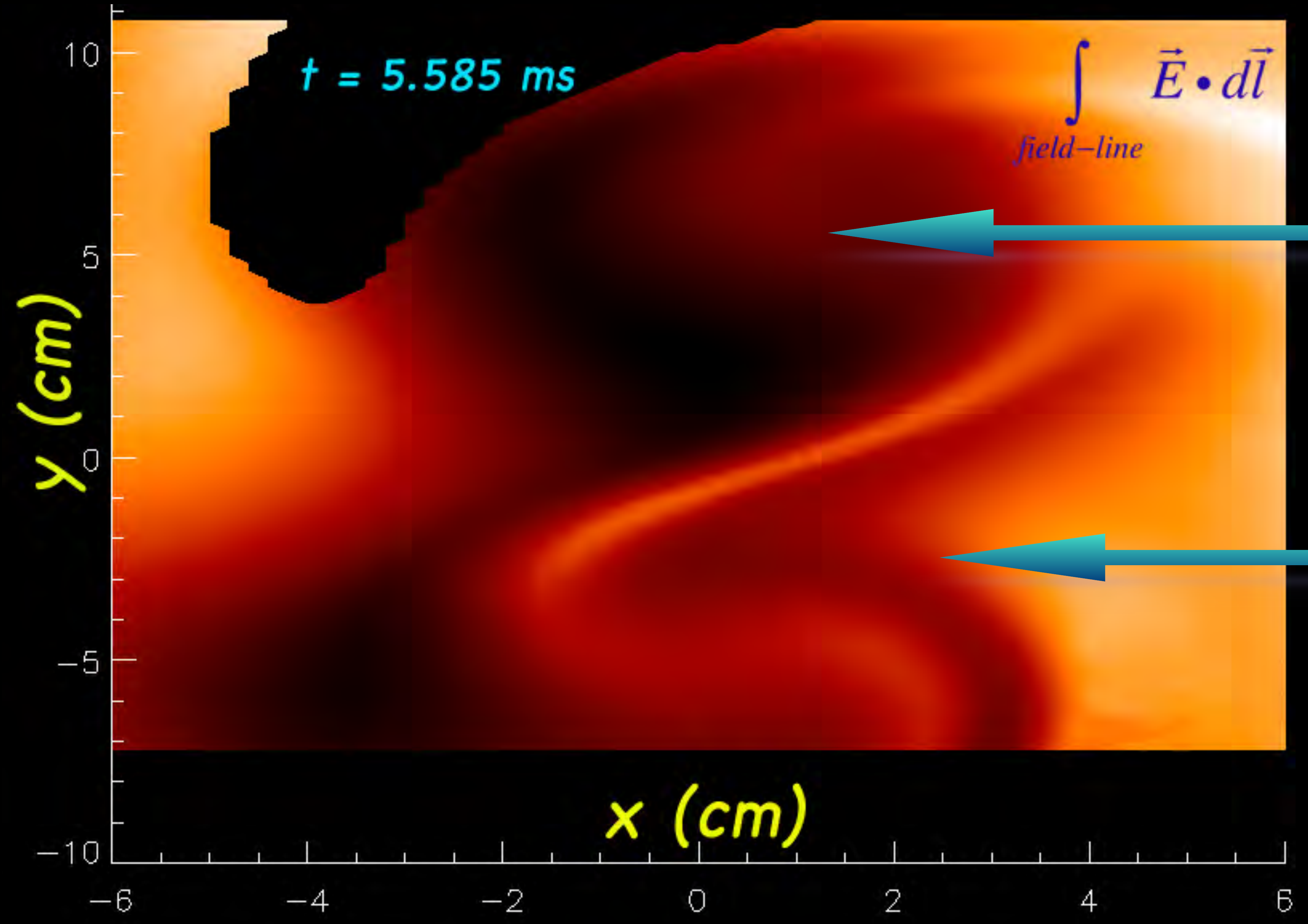
Reconnection Rate

$$\Xi = \int_{field-line} \vec{E} \bullet d\vec{l}$$



$$\sigma_{\nu\mu}(\omega,\vec{r}) = \frac{ne^2}{m v_{the}^2} \int e^{-i\omega t} v_\nu(t) v_\mu(t+\tau)\, dt d\tau$$

Kubo AC resistivity $\omega = 2\pi f_{rope}$

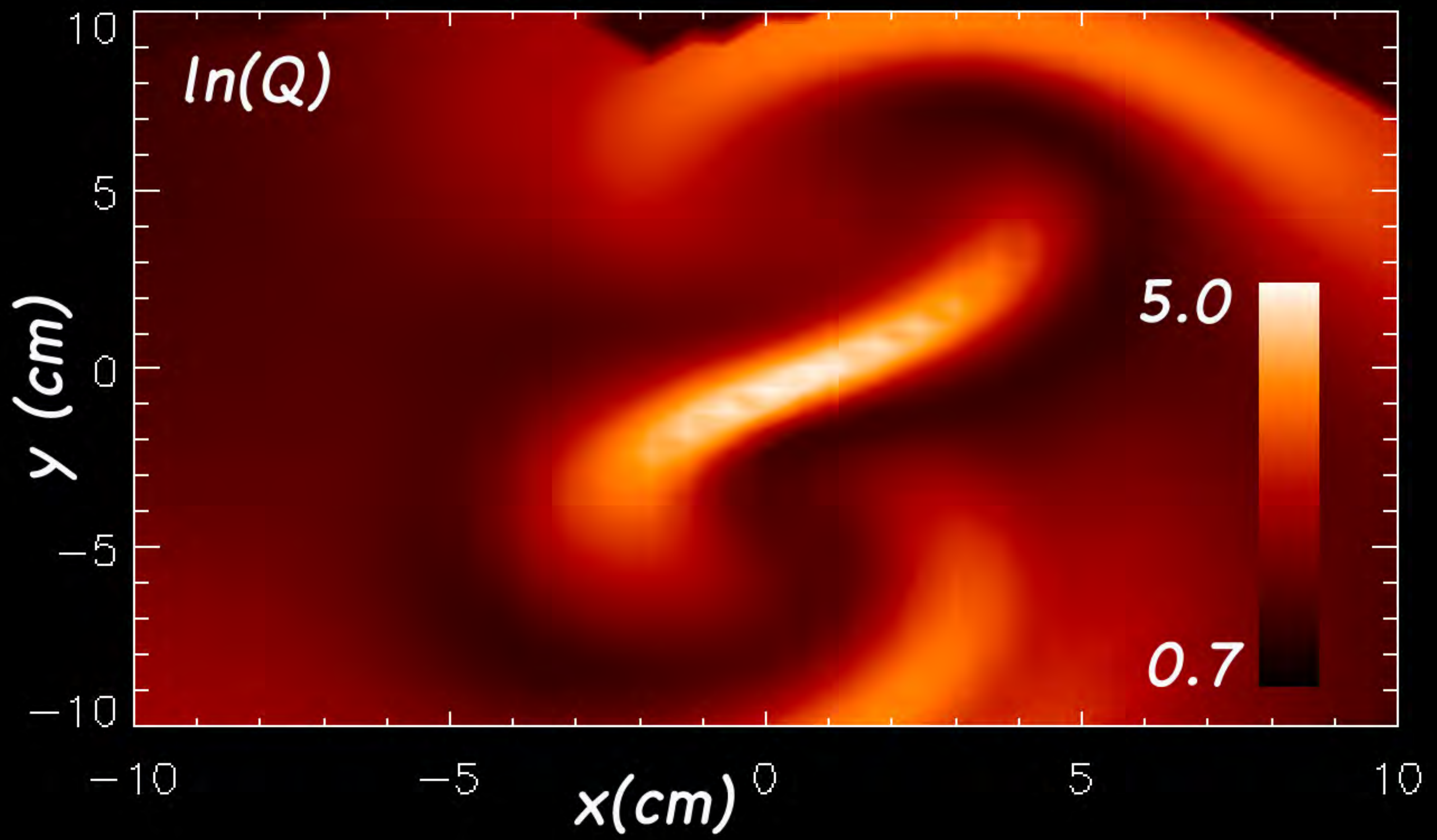


−5 V

reconnection rate

+5 V

t = 5.79 ms



$$\int_{all-fiellines} \vec{E} \bullet d\vec{l} \simeq -.75V \pm 20\%$$

Phys. Rev Lett., **116**, 235101 (2016)

Alfvénic Reconnection Rate

$$R_n = \frac{\Xi}{L B_\theta V_A}$$

L = 11 m

B_θ = 10 G ; n = 4.0X10^{12} cm^{-3} ; V_A = 1.8X10^5 m/s

$$R_n \simeq 0.1$$

W. Gekelman et al Phys. Rev. Letts, 116, 235101 (2016)

Two interacting flux ropes:

Twist and writhe about themselves , wrap around each other

Collide when they are kink unstable

Magnetic field line reconnection occurs at each collision

Ohms law for flux ropes is non-local

The resistivity can be deduced using the Kubo theory

Changes in flux rope helicity can also be used to derive $\langle \eta_{\|} \rangle$

Flux ropes are chaotic

Topological Approach

Calculate positions of many field lines vs time

Use field lines to calculate winding number

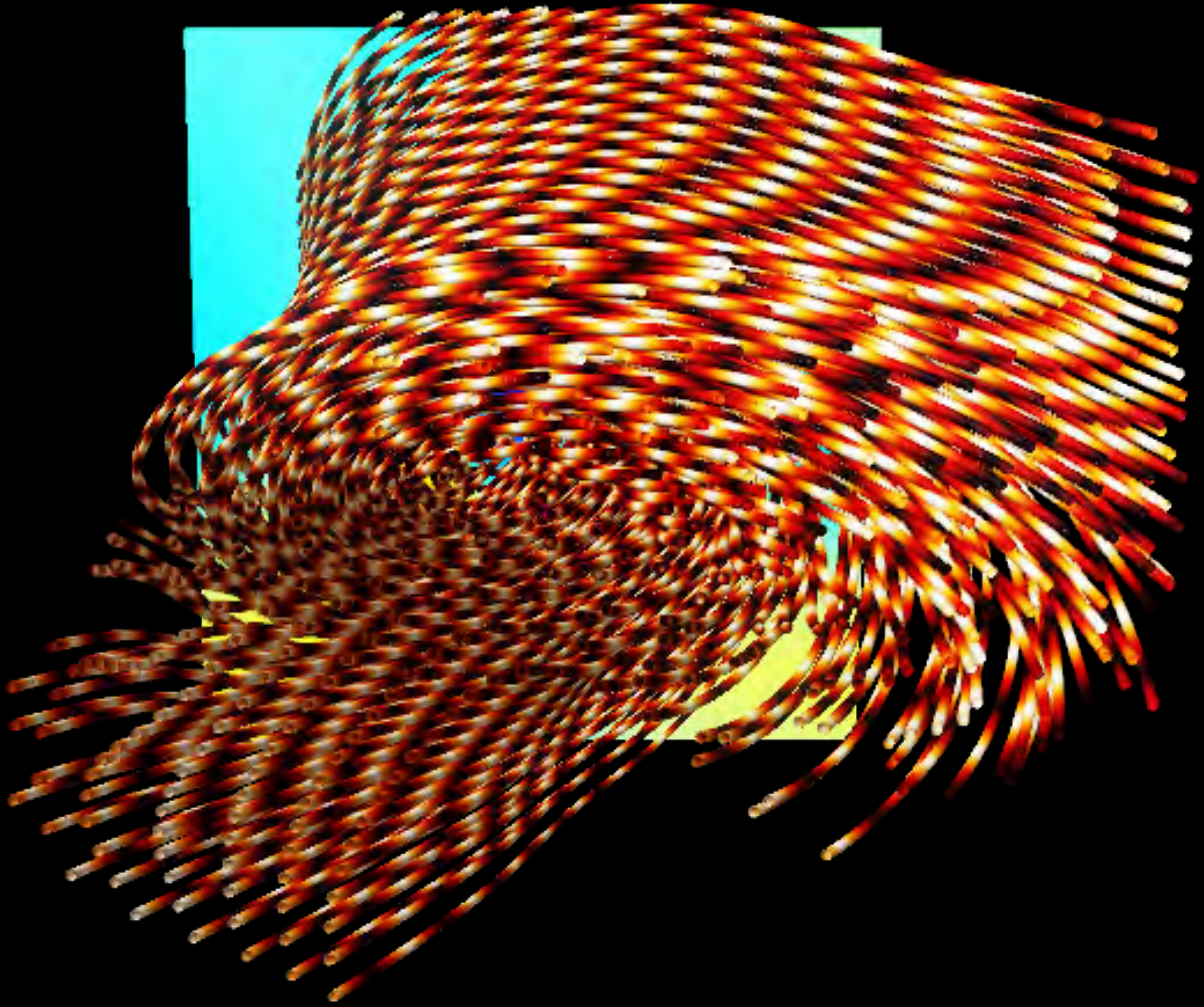
Correct winding number for boundary motion
and ideal flow

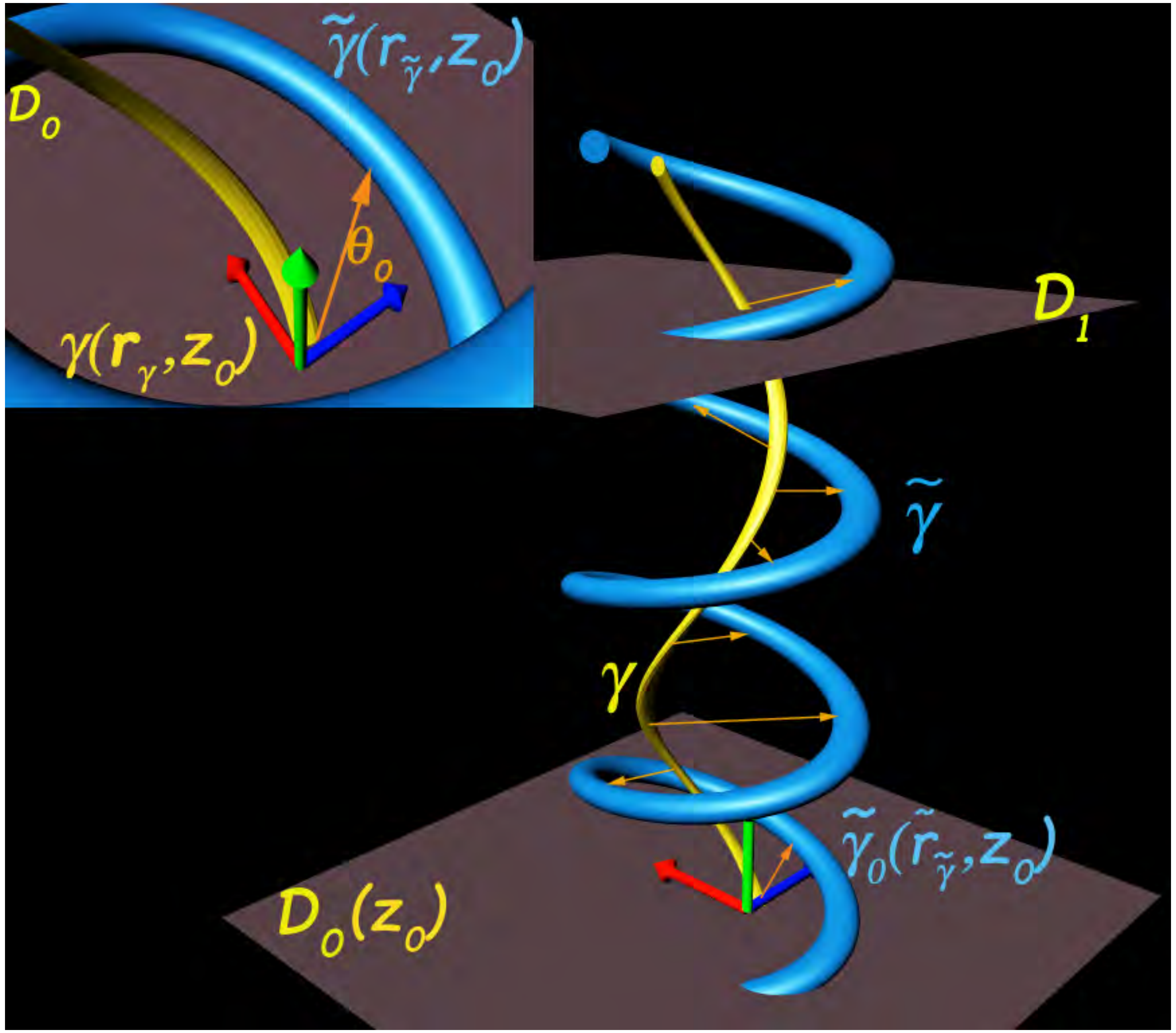
Calculate Helicity corrected for ideal flow

Use these to calculate "reconnective activity"

Magnetic Field Lines

t = 2.953 ms





Winding Angle

$$\Theta(\tilde{\gamma},\gamma,z) = \arctan\left(\frac{\tilde{\gamma}_2(\vec{r}_{\tilde{\gamma}},z)-\gamma_2(\vec{r}_\gamma,z)}{\tilde{\gamma}_1(\vec{r}_{\tilde{\gamma}},z)-\gamma_1(\vec{r}_\gamma,z)}\right)$$

$\vec{r}_\gamma$ position of test field line in plane z

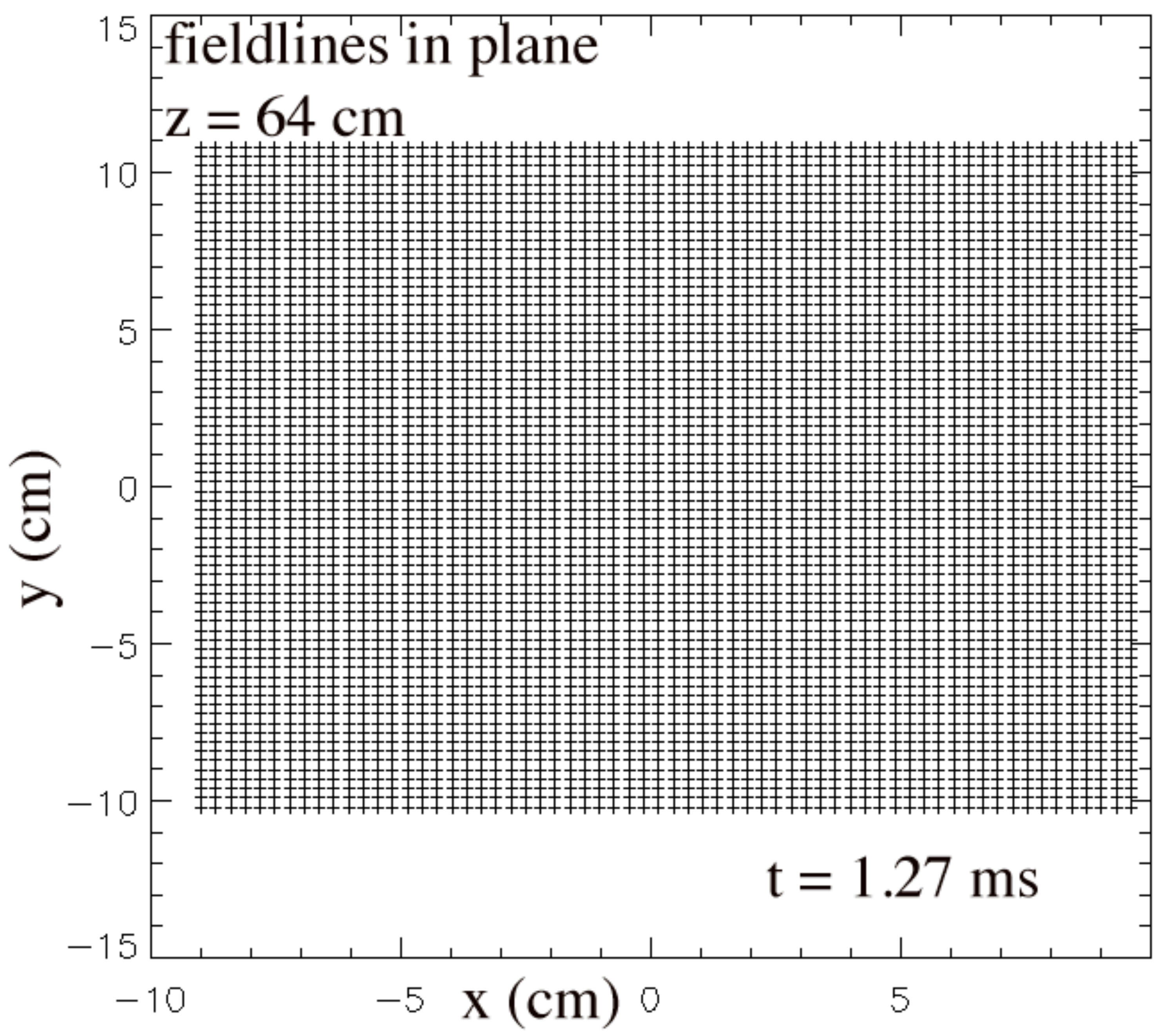
$\vec{r}_{\tilde{\gamma}}$ position of all other field lines in plane z

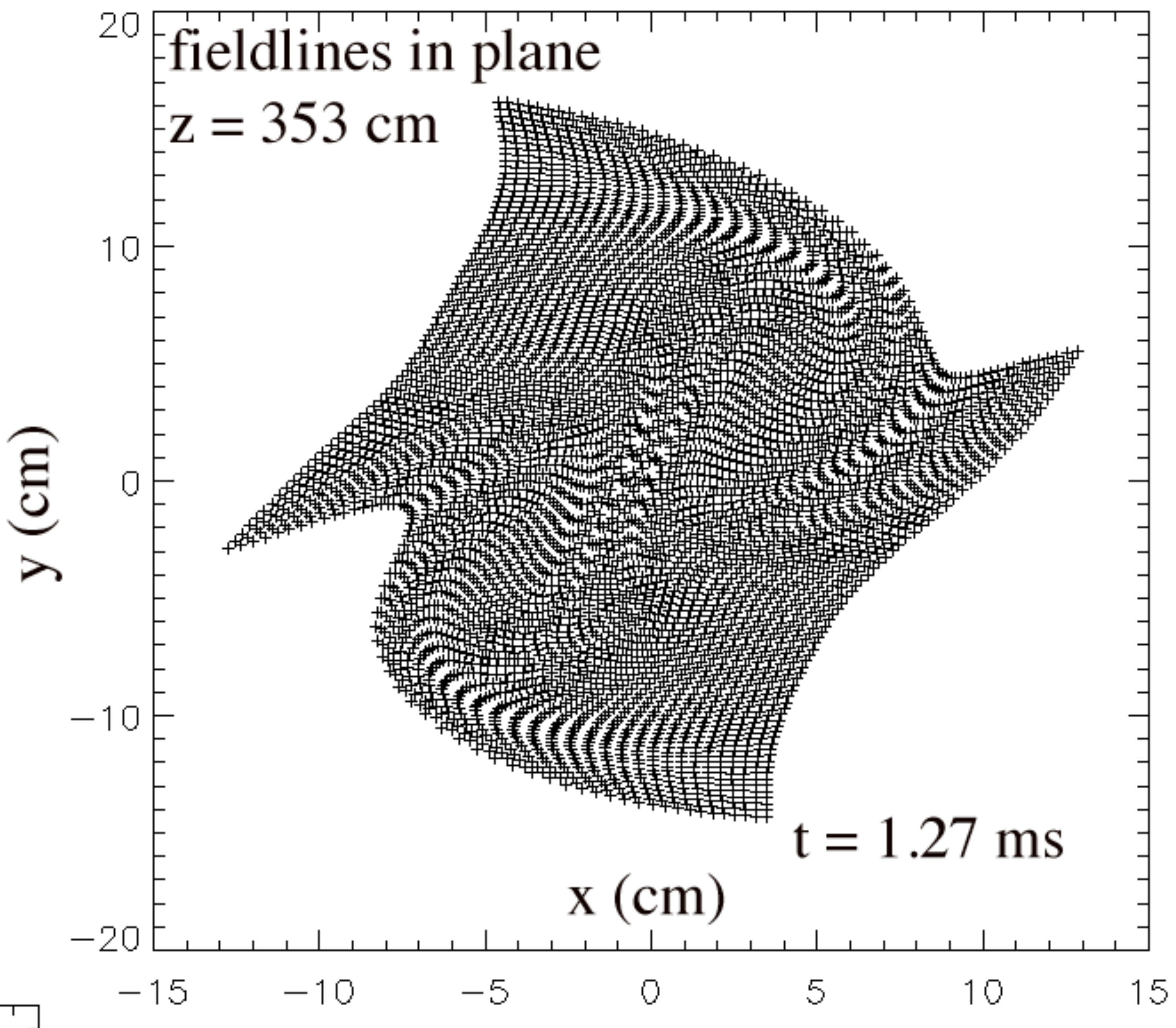
γ_2 = y position of test field line at $\mathbf{r}_\gamma$

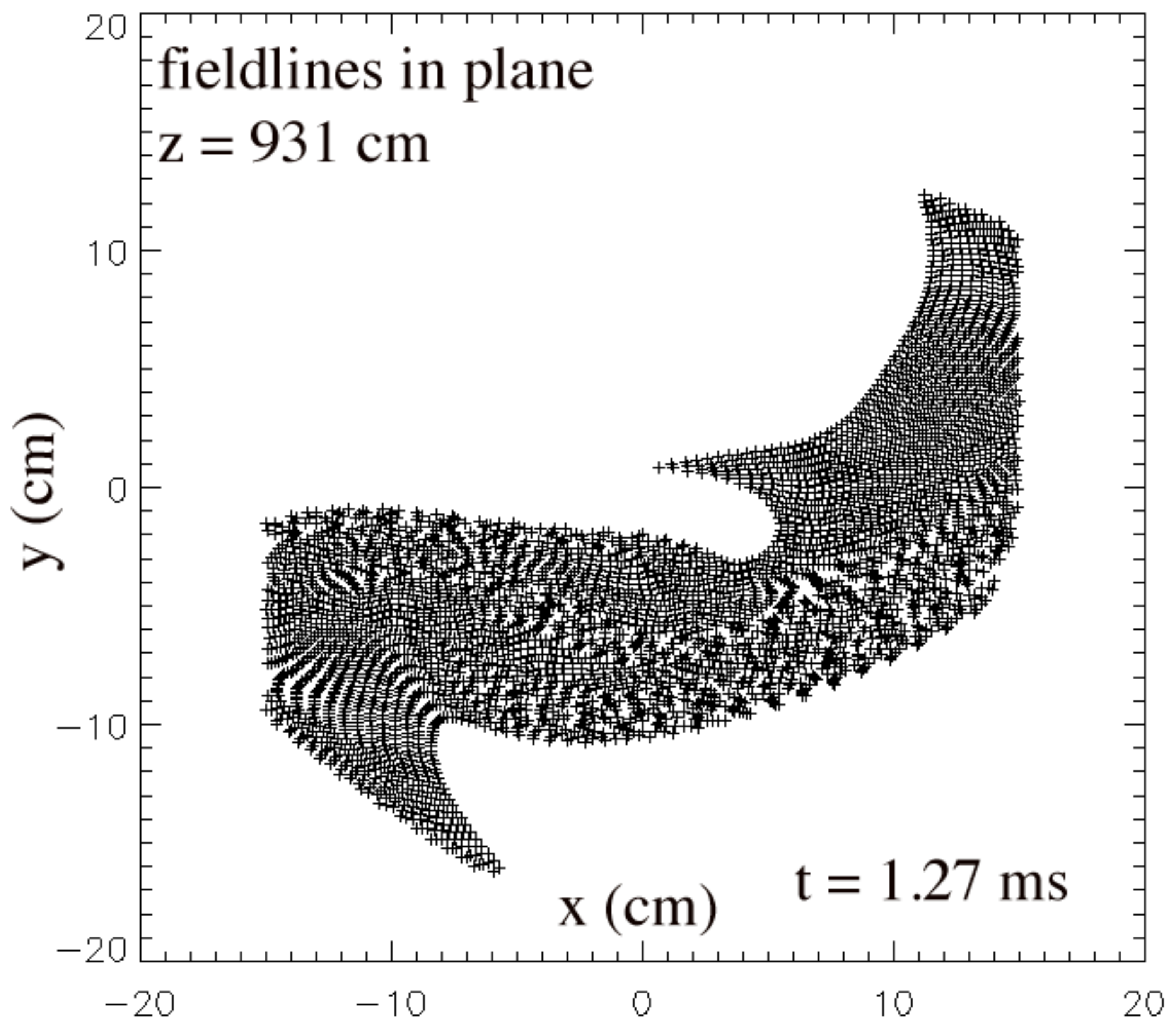
γ_1 = x position of test field line at $\mathbf{r}_\gamma$

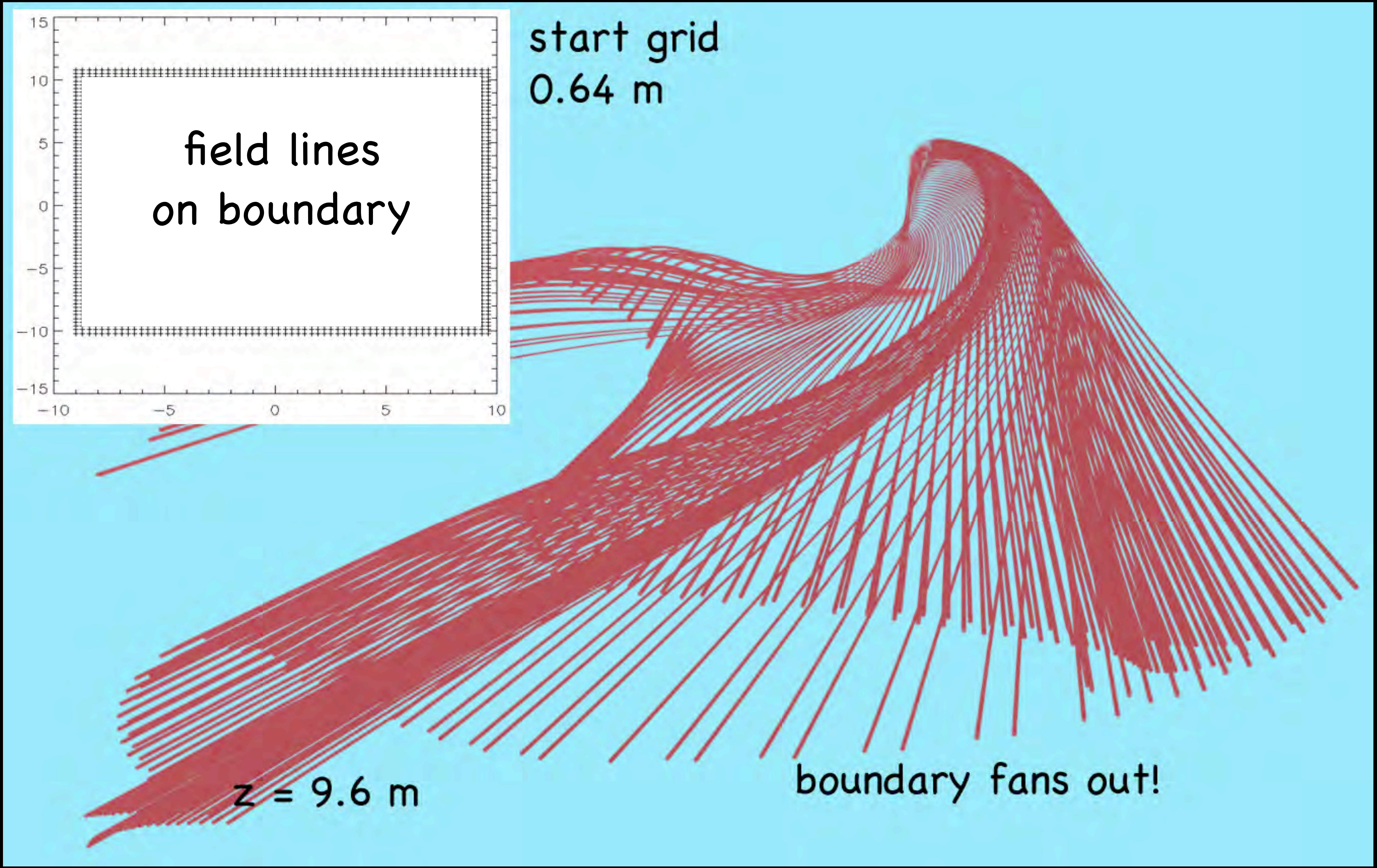
Field lines used to calculate

$$\Theta(\gamma,\tilde{\gamma},z)$$

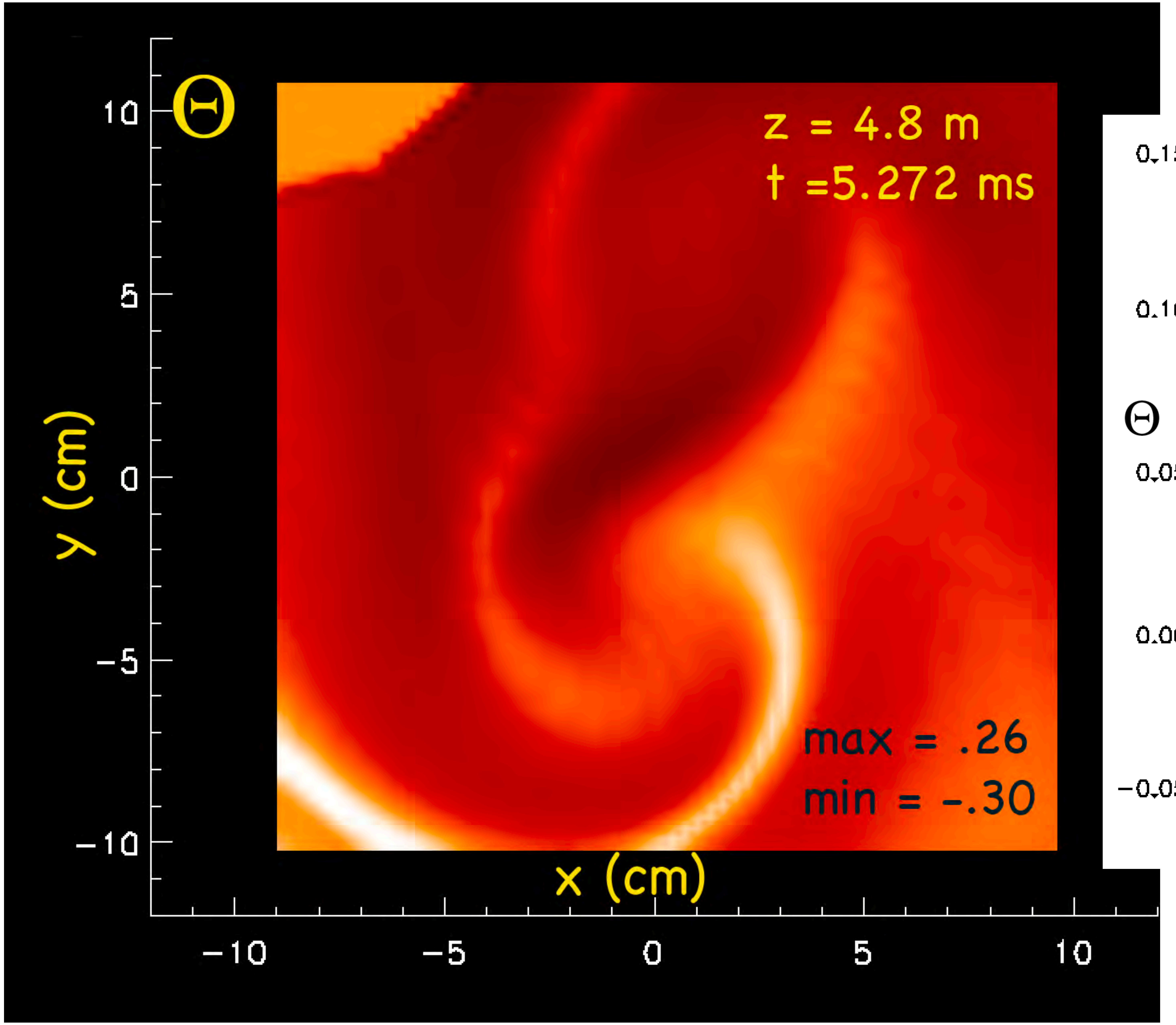




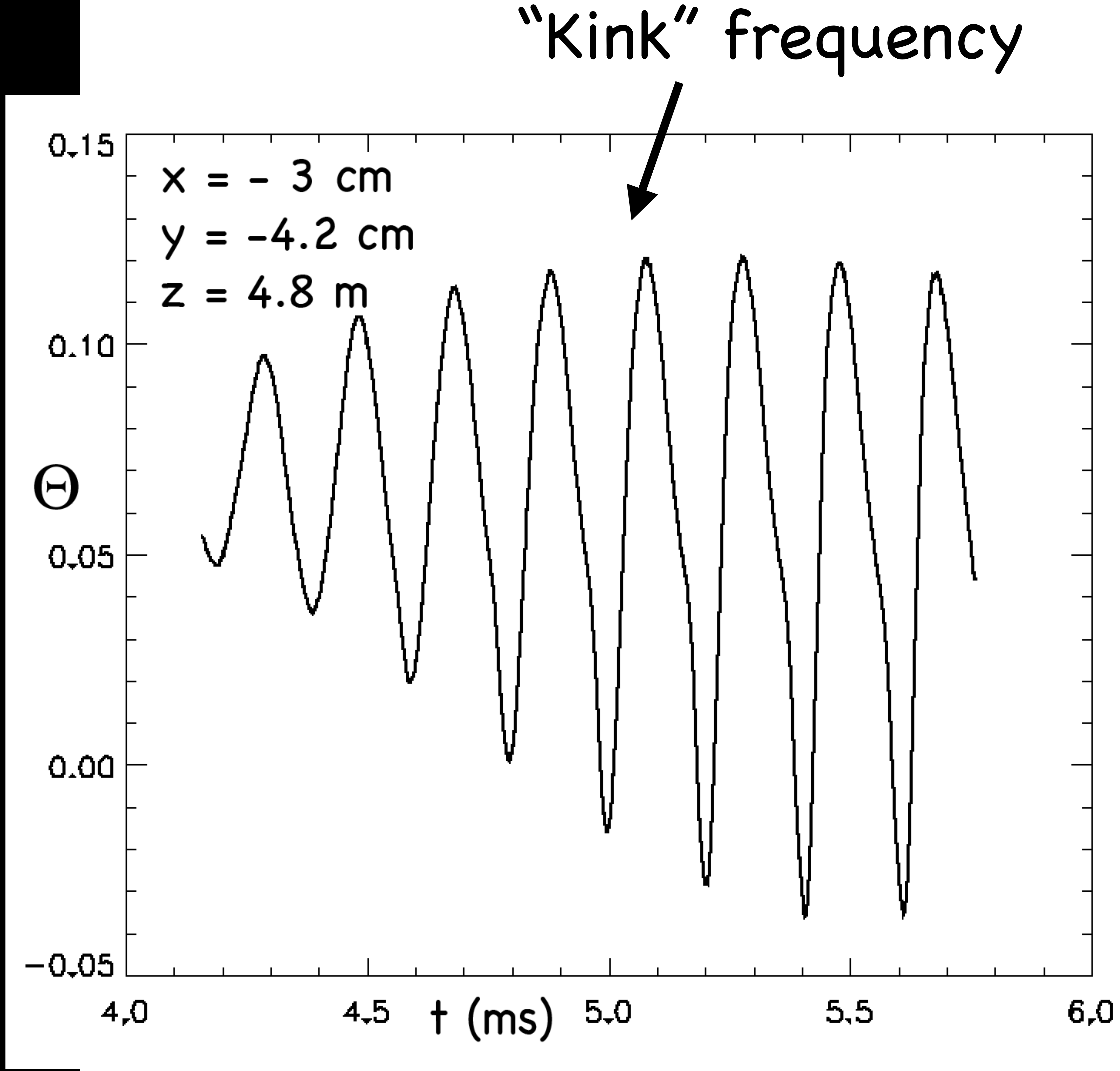



start grid
0.64 m
field lines
on boundary
15
10
5
0
−5
−10
−15
−10
−5
0
5
10
z = 9.6 m
boundary fans out!

Θ (z = 4.8 m)   Winding Number



Θ(t)



L is mean entanglement over all field lines

If there are no branch cuts ie $\Theta\left(\tilde{\gamma},\gamma,z\right) \leq 2\pi$

$$L\left(\vec{r}_0,z,t\right)=\frac{1}{2\pi}\int_{D_0(t)}\left[\Theta\left(\vec{r}_0,\vec{r},z,t\right)-\Theta\left(\vec{r}_0,\vec{r},0,t\right)\right]dA$$

Next correct for boundary motion

1) Take each field line used to calculate Θ at t-dt

2) Assume filed lines are frozen and in time dt (here 300ns) move the field lines such that:

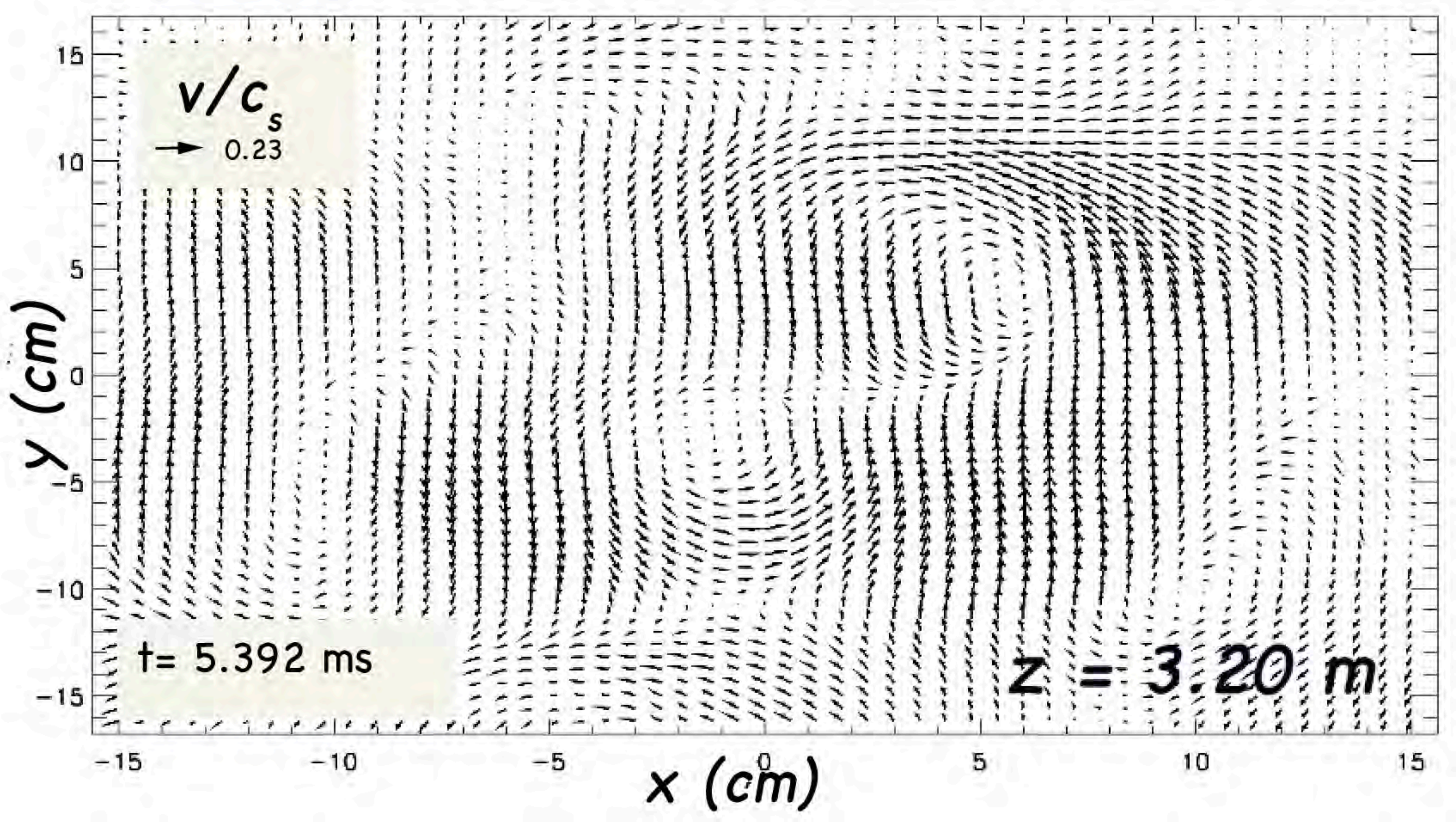
$$\vec{r}_{fieldline}(t) = \vec{r}_{fieldline}(t - dt) + \vec{u}dt \qquad \vec{\mathrm{u}} = \text{ion flow}$$

3) Recalculate Θ

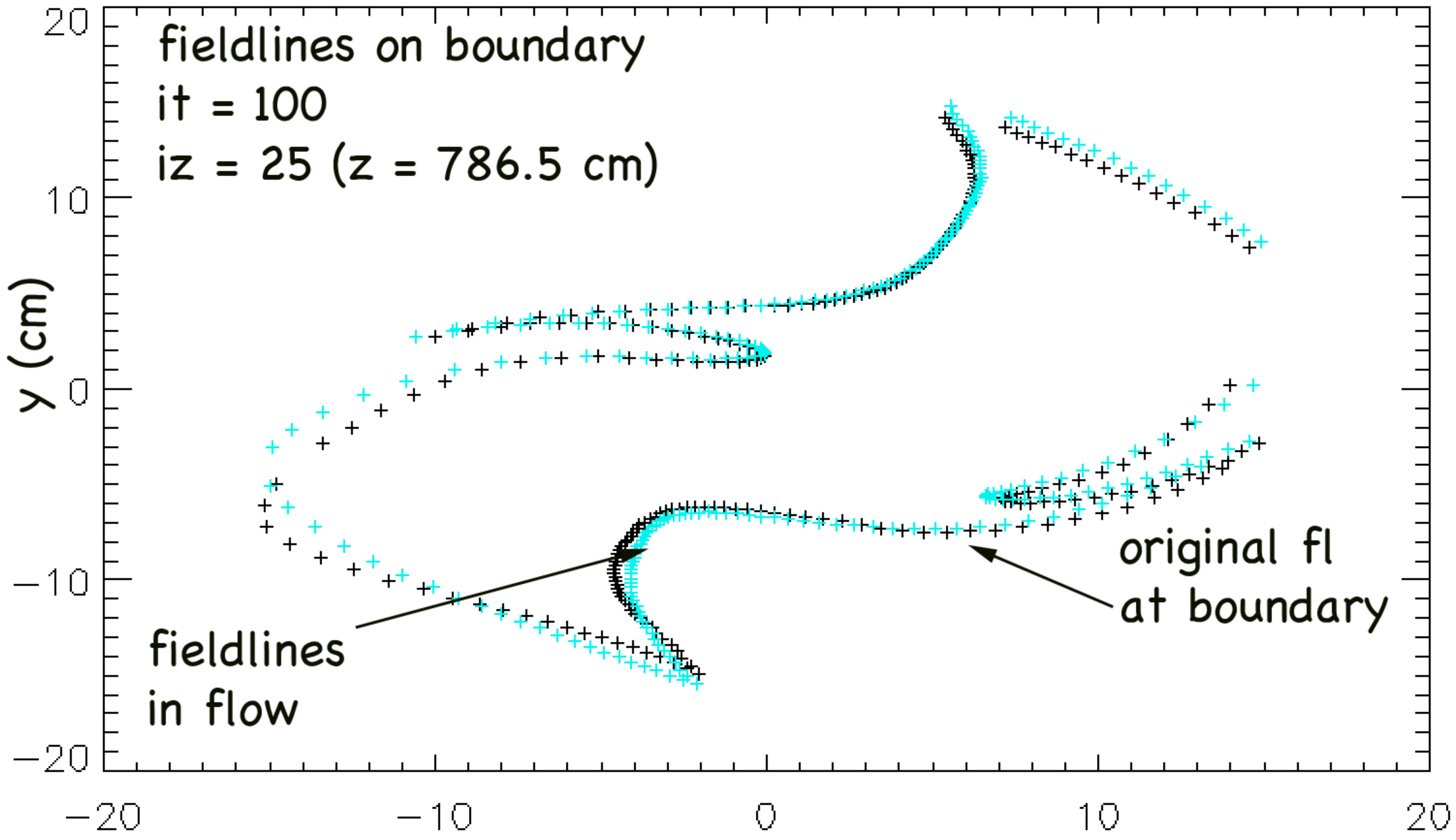
$$\Delta L_{ideal}\left(\vec{r}_0, z, t, dt\right) = \frac{1}{2\pi}\int_{D_0(t)}\left[\Delta\Theta_{vel}\left(\vec{r}_0, \vec{r}, z, t\right) - \Delta\Theta_{vel}\left(\vec{r}_0, \vec{r}, 0, t\right)\right]dA$$

$$L_{total}\left(\vec{r}_{\gamma},t\right)=(L\left(\vec{r}_{\gamma},t\right)-L\left(\vec{r}_{\gamma},t-dt\right)-\Delta L_{ideal}\left(\vec{r}_{\gamma},t\right)$$

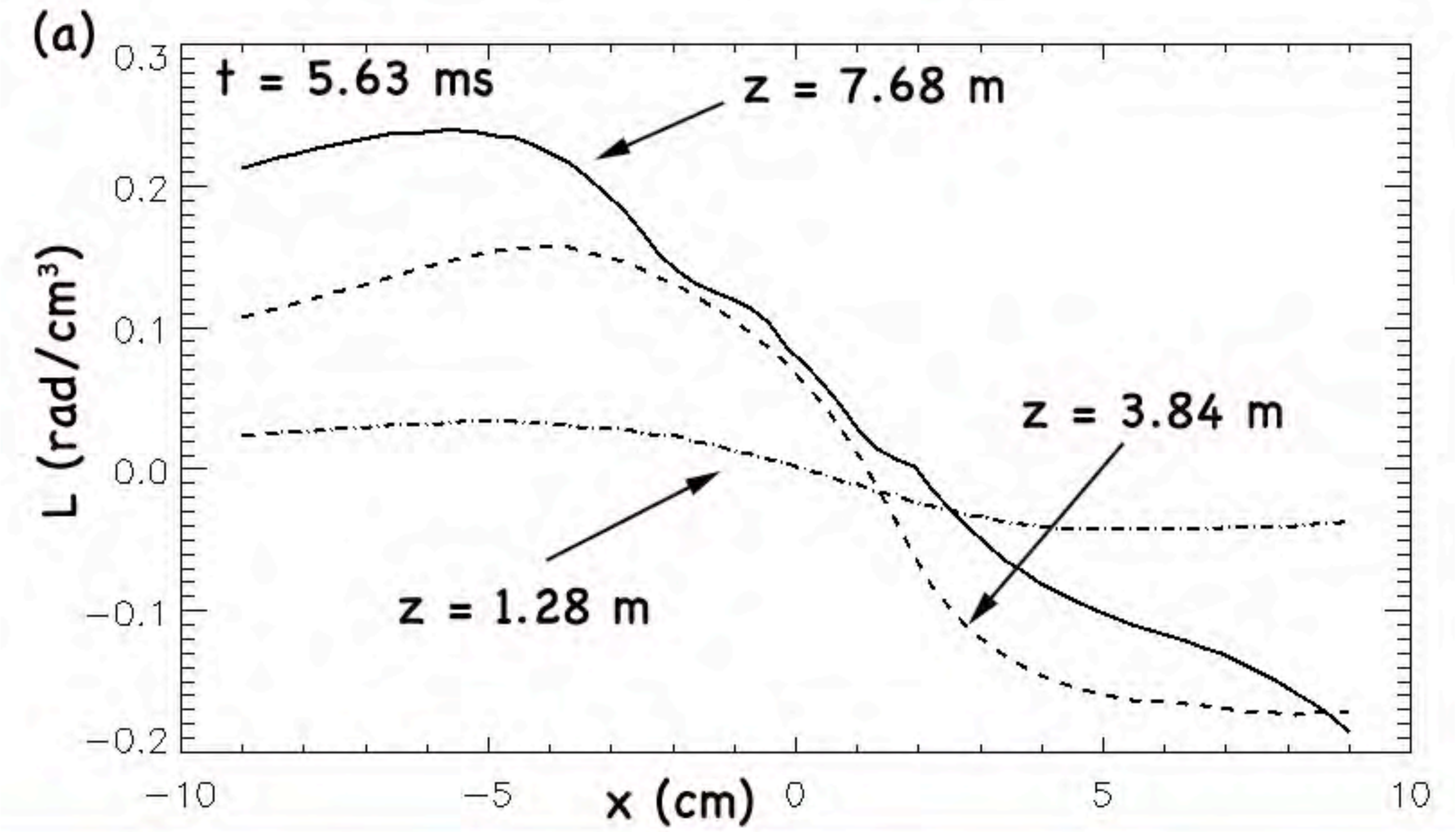
Plasma Flow



Field Line Boundary Motion Due to Flow

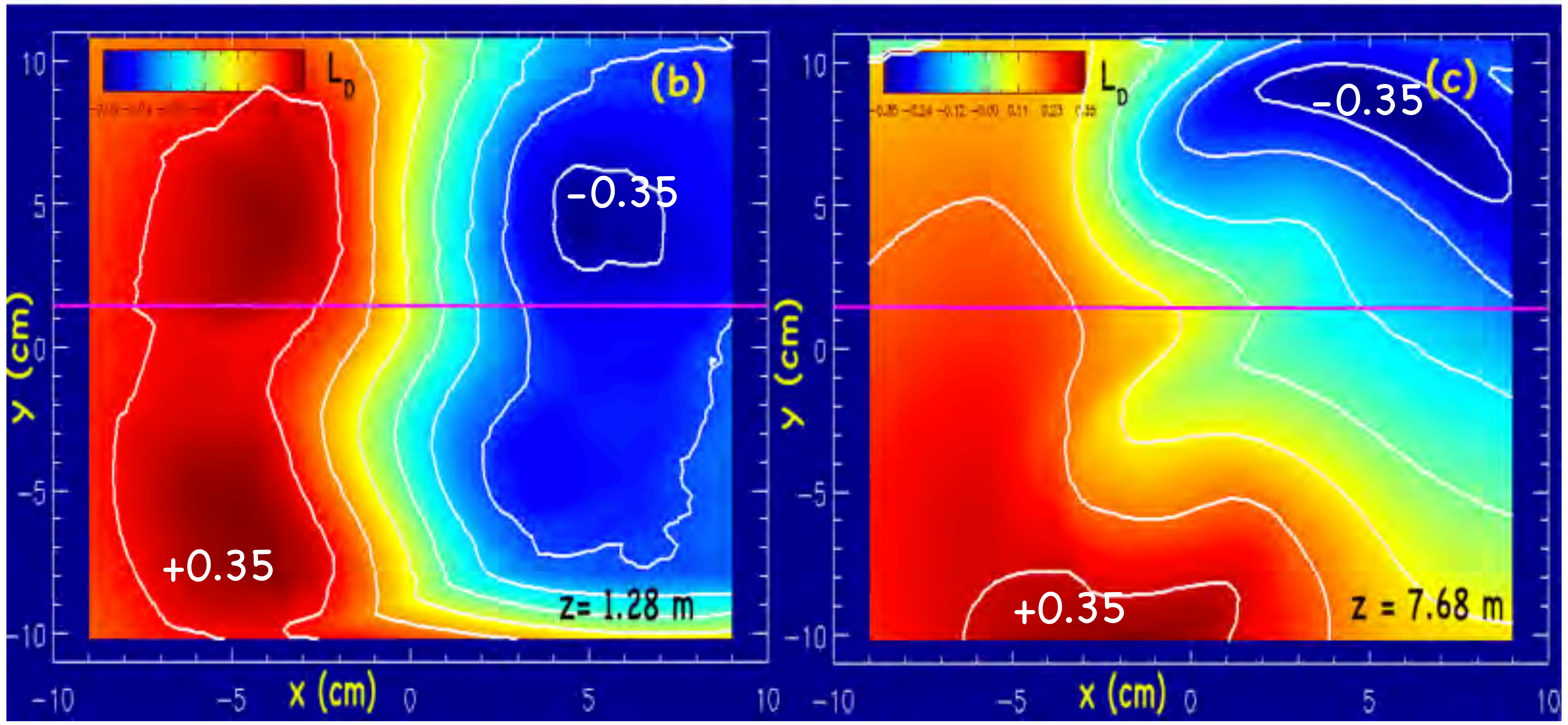


$L_{\text{boundry motion}}$ term is 1-2% of L and is ignored



Winding number density L_D

t = 5.63 ms
x=0
y = 1.6 cm



z = 1.28m

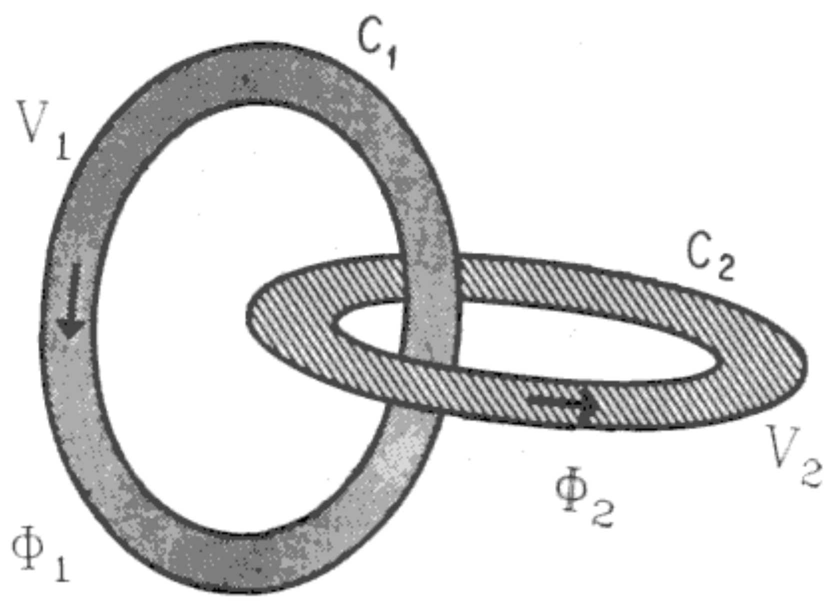
z = 7.68 m

Winding number density L_D on two planes

-10 cm < y , 10 cm

Helicity was shown to be the product of dΘ/dt and axial B field

$$H = \int \vec{A} \bullet \vec{B} d^2 \vec{r}_\gamma dz$$



$$H = \phi_1 \phi_2$$

units:
Volts*time/Area

helicity is related to winding number*, L

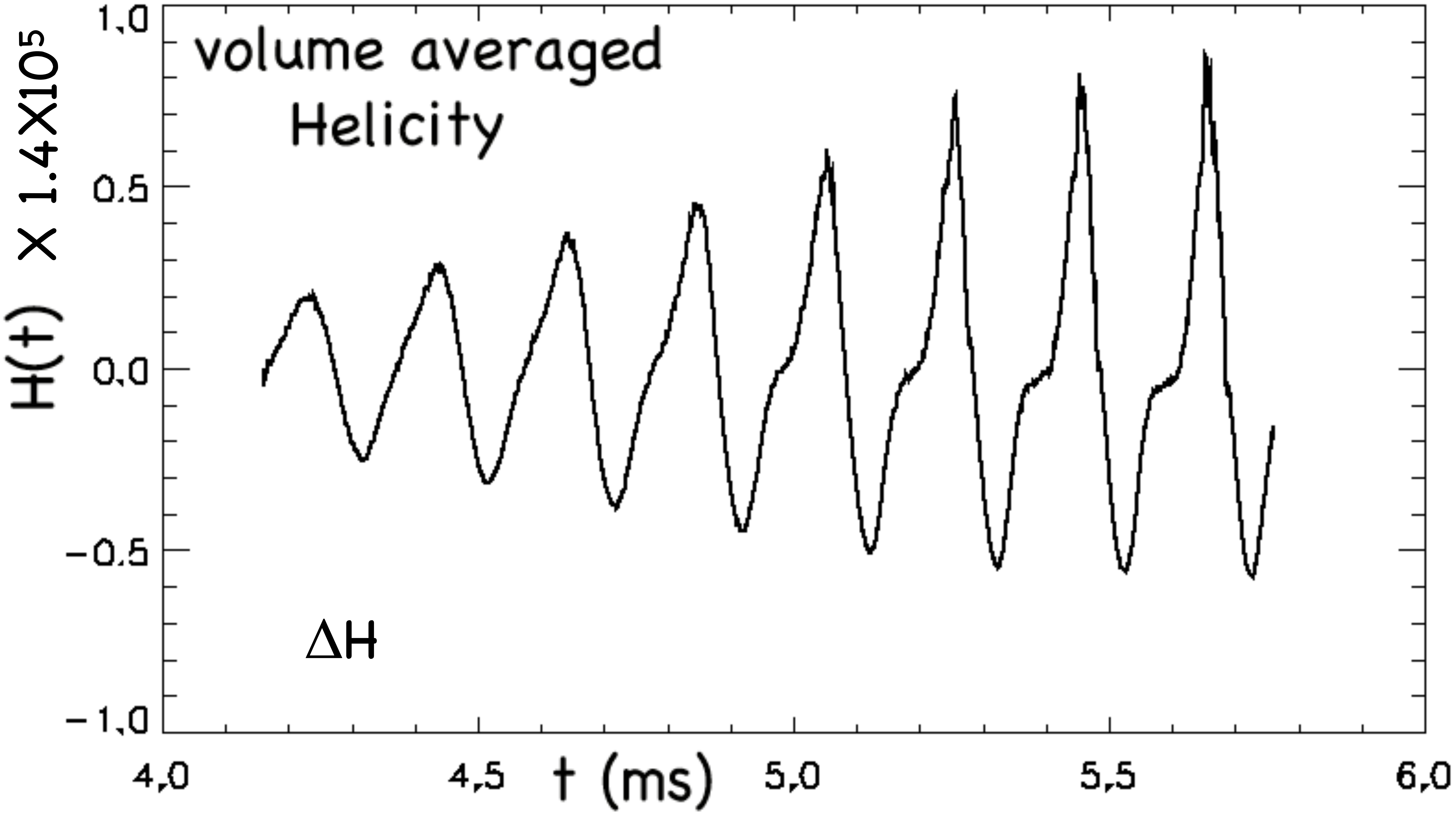
$$\Delta H\left(\vec{r}_0, z, t\right) = \frac{1}{2\pi} \int_{D_0(t)} B_z\left(\vec{r}, t\right) \frac{\partial}{\partial t}\left[\Theta\left(\vec{r}_0, \vec{r}, z, t\right) - \Theta\left(\vec{r}_0, \vec{r}, 0, t\right)\right] dA \quad ; \mathrm{dA} = \mathrm{d}^2 \vec{r}_\gamma$$

*Prior and Yeates, ApJ (2014), Berger, Plasma. Phys. Control. Fusion (1999)

Helicity Density

t = 5.513 ms

t = 5.612 ms


volume averaged
Helicity
H(t) X 1.4X10^5
1.0
0.5
0.0
−0.5
−1.0
ΔH
4.0
4.5
t (ms)
5.0
5.5
6.0

t = 5.6285 ms
z = 9.6 m
z = 4.8 m
z = 2.4 m
L
1.0
0.5
0
-0.5
-1.0
Q = 100
J(z=0.64 m)

Reconnective Activity

Winding number density

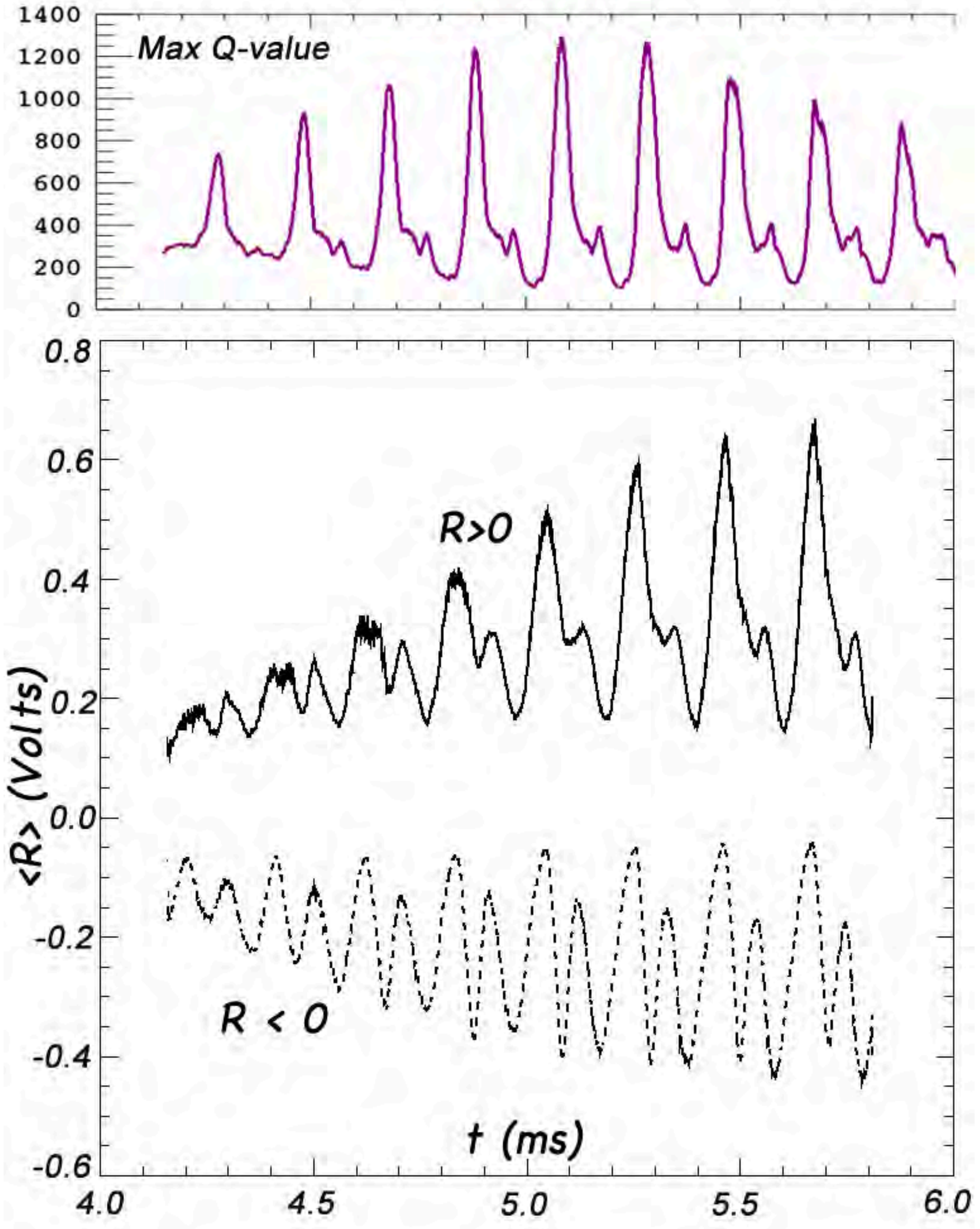
ΔL_{ideal} is change of L due to flow over time dt

$$R\left(\vec{r}_\gamma, t\right) = \frac{L\left(\vec{r}_\gamma, t\right) - L\left(\vec{r}_\gamma, t - dt\right) - \Delta L_{ideal}\left(\vec{r}_\gamma, t\right)}{dt} \Delta H$$

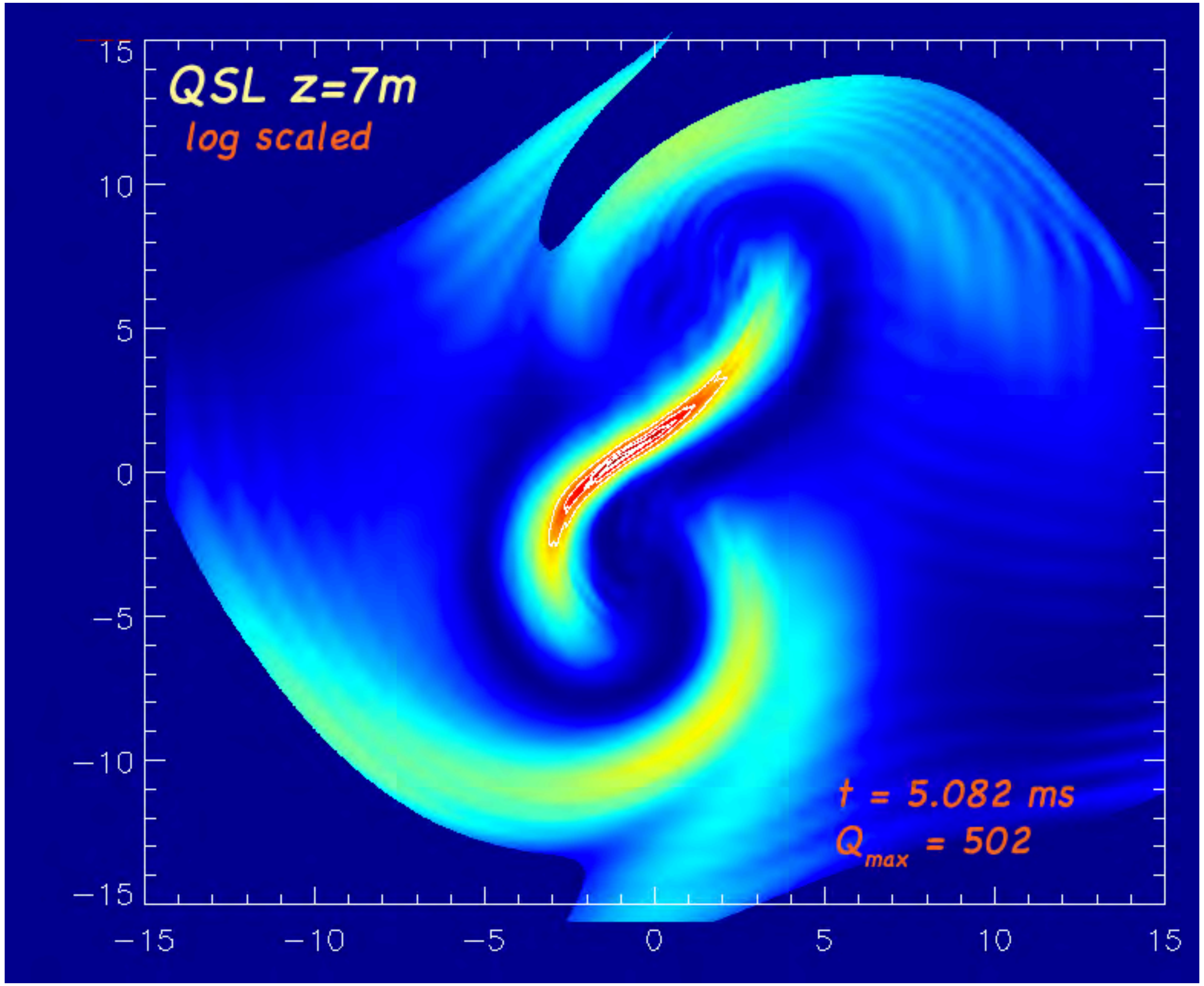
Units in Volts

helicity

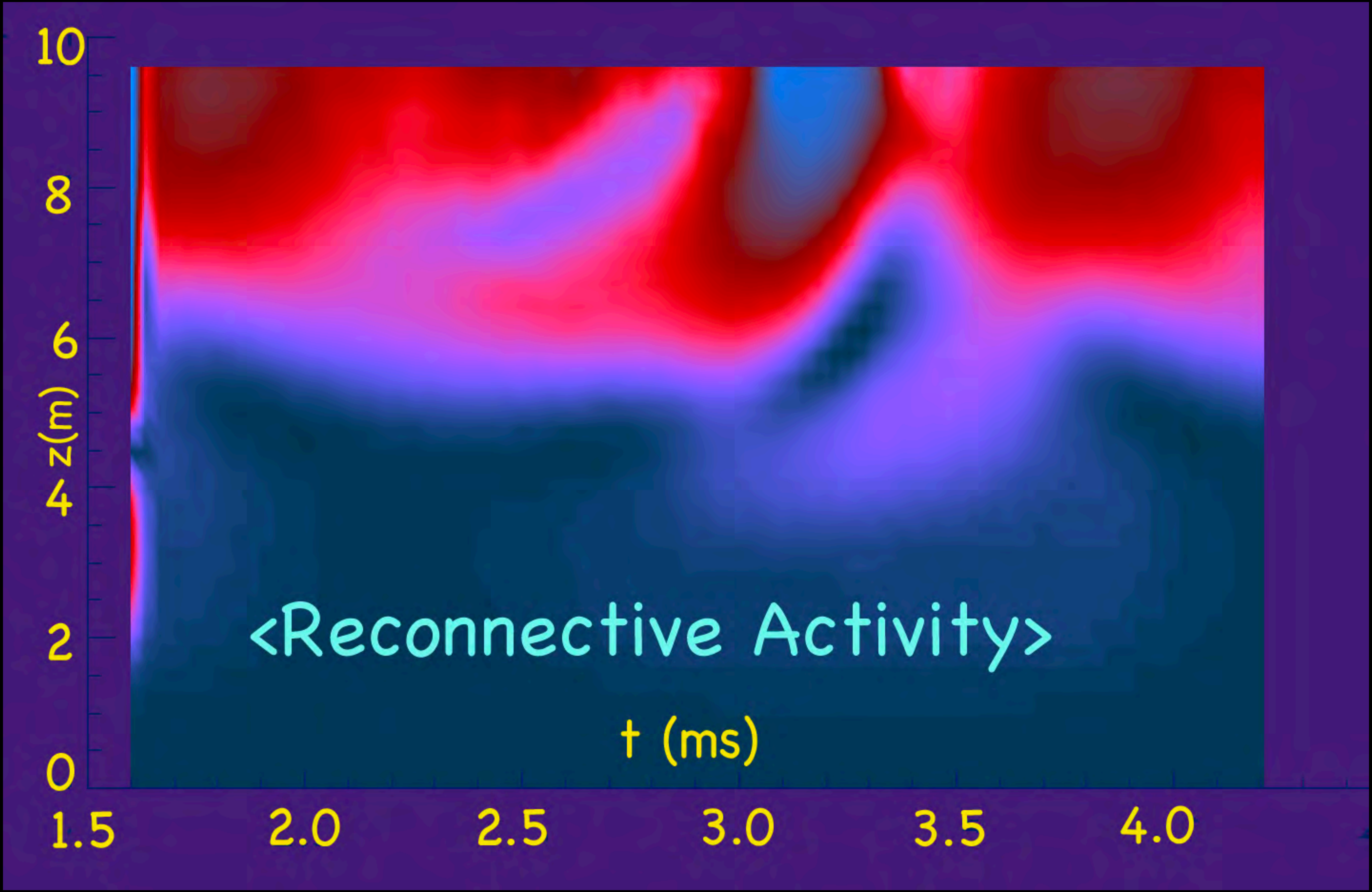
Reonnective activity
R = -2.5 V
Q = 100
z= 9.6 m
z = 7 m
R
4.59 V
0
-3.91 V
R = 2.5 V
δt = 3.0695 ms
z = 0.64 m



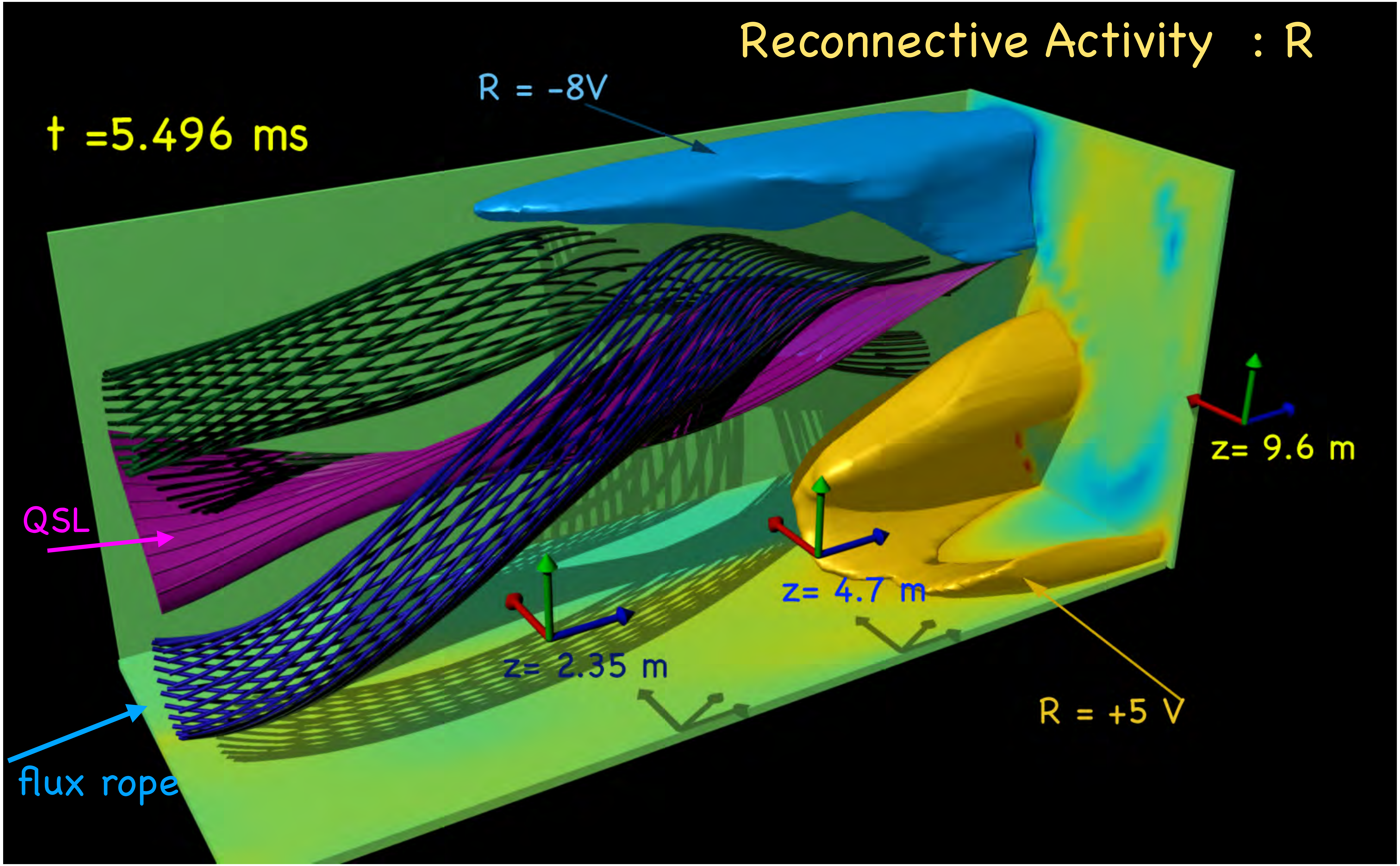
Max Q-value
1400
1200
1000
800
600
400
200
0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
<R> (Volts)
R>0
R < 0
t (ms)
4.0
4.5
5.0
5.5
6.0



QSL z=7m
log scaled
t = 5.082 ms
Q_{max} = 502
15
10
5
0
−5
−10
−15
−15
−10
−5
0
5
10
15



<Reconnective Activity>
z(m)
10
8
6
4
2
0
t (ms)
1.5
2.0
2.5
3.0
3.5
4.0


Reconnective Activity : R
t =5.496 ms
R = -8V
QSL
flux rope
z= 2.35 m
z= 4.7 m
z= 9.6 m
R = +5 V

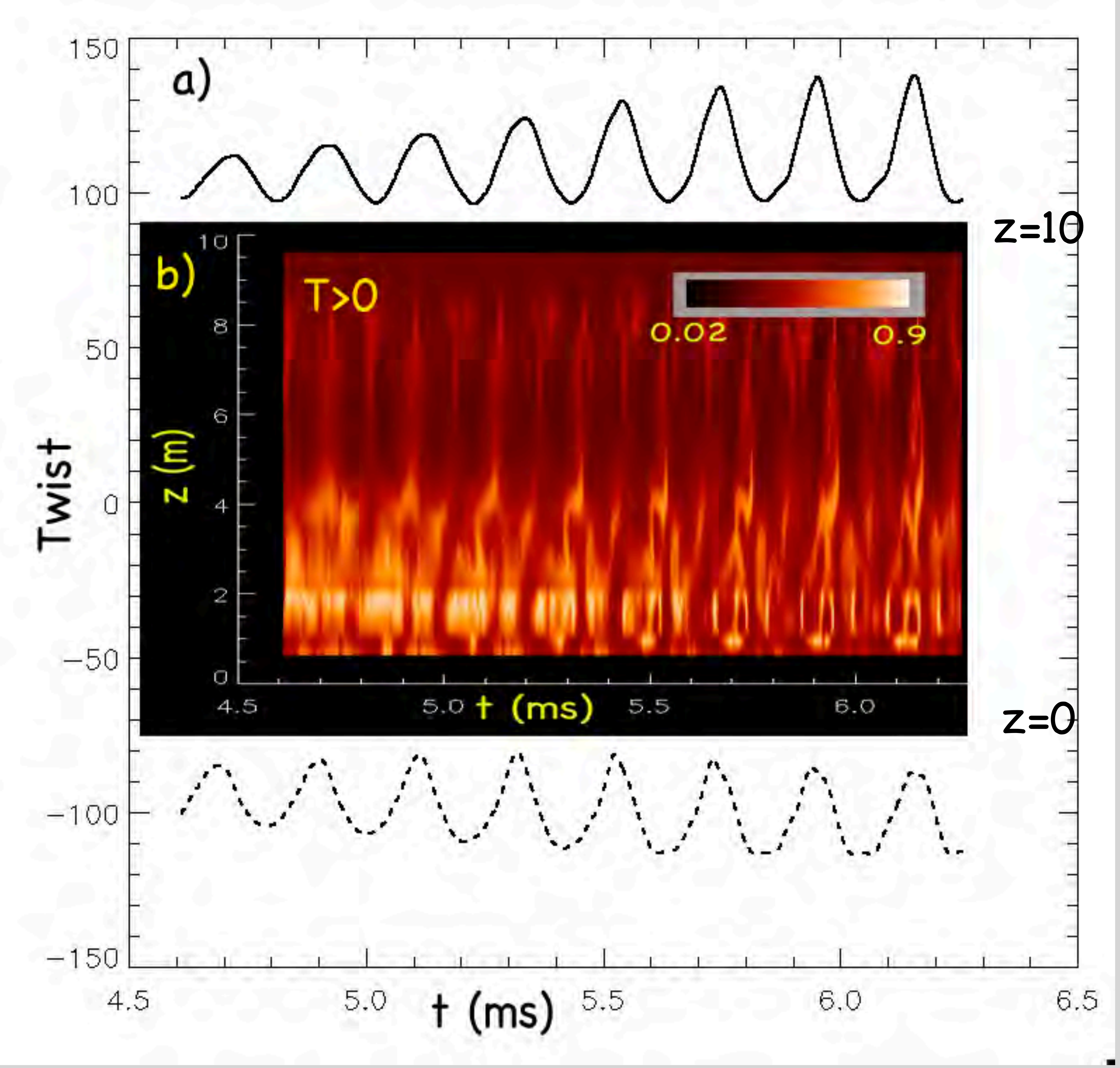
Twist

Amps/Tesla-m³

$$\boxed{T(\vec{r},t) = \int_{all\ \gamma(\vec{r})} \frac{\vec{J}\bullet\vec{B}}{B^2} d^3r}$$

integration over all field lines

T (Amps/Tesla)
area integrated twist density $\int_{area} T(x,y,z,t)\,dx,dy$



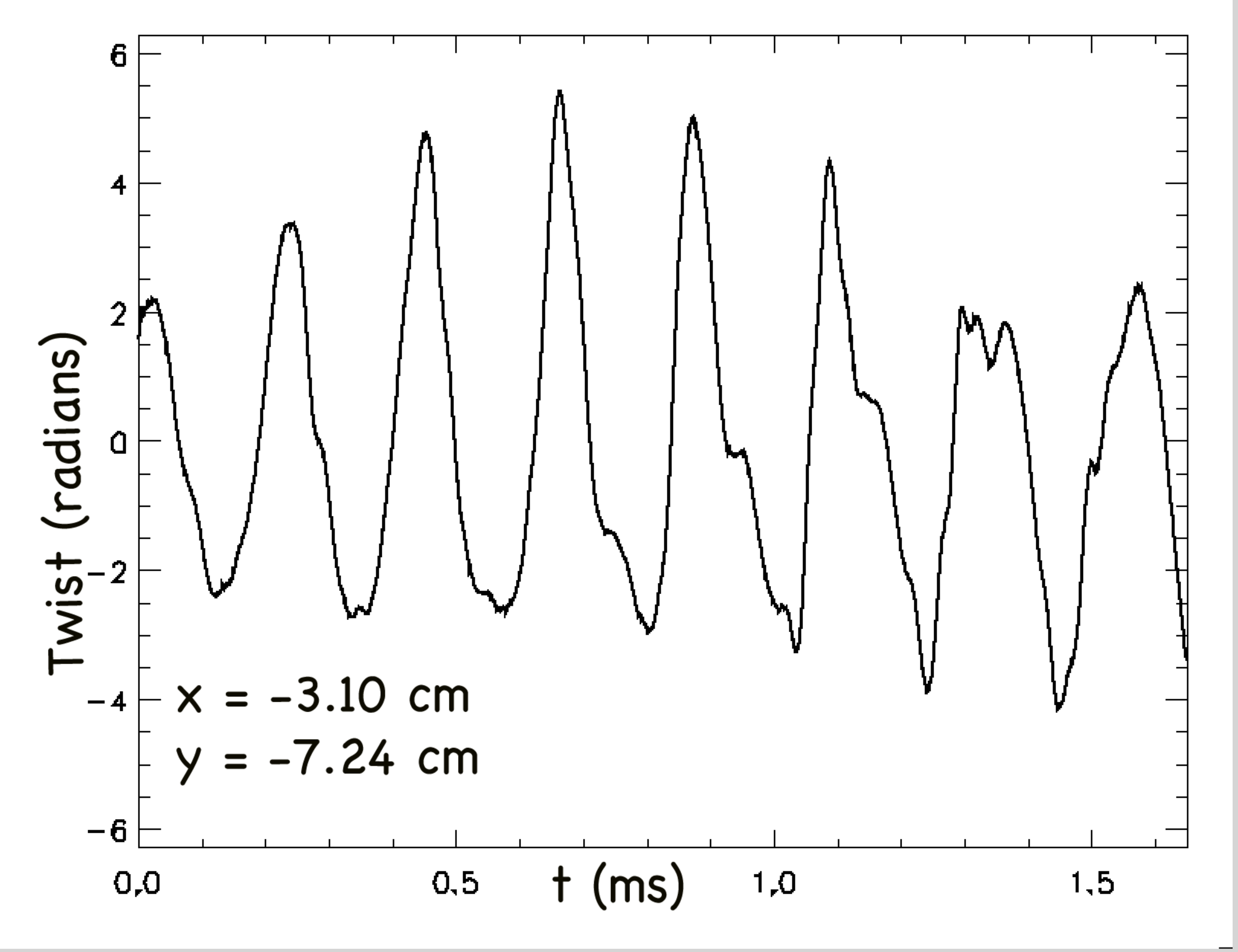
positive/negative twists separated

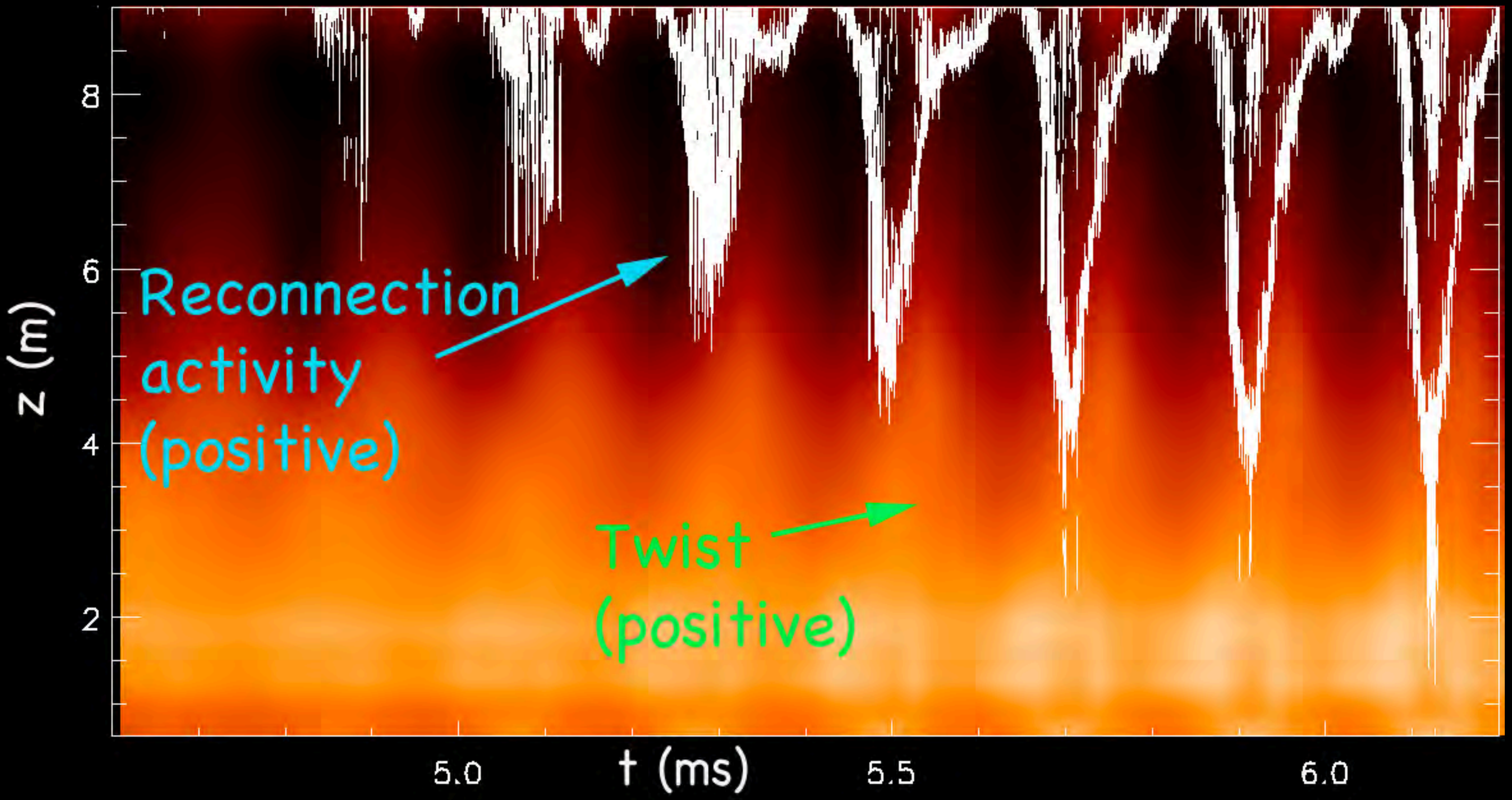
Twist

Amps/Tesla-m³

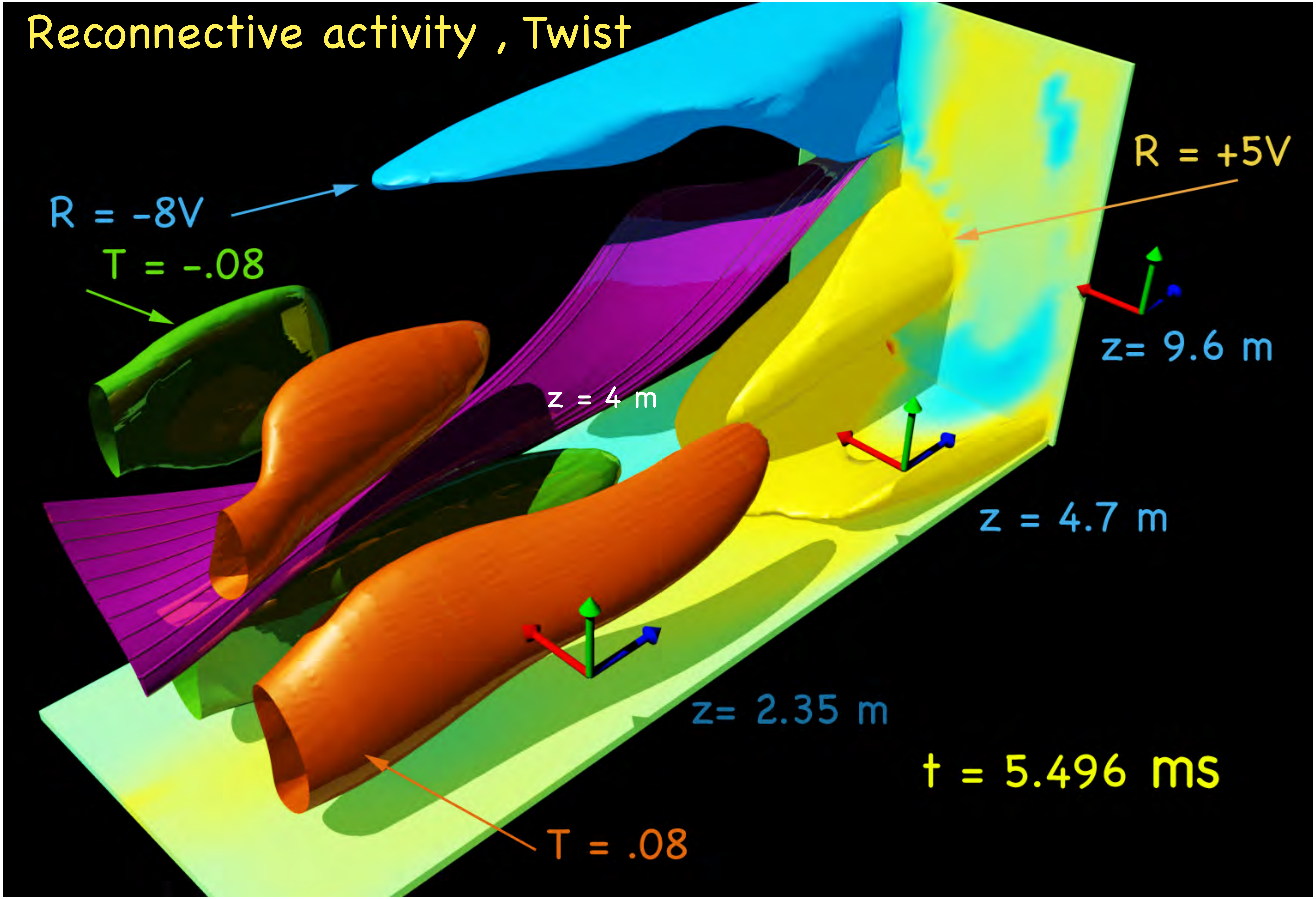
$$\boxed{T(\vec{r},t) = \int_{all\ \gamma(\vec{r})} \frac{\vec{J}\bullet\vec{B}}{B^2} d^3r}$$

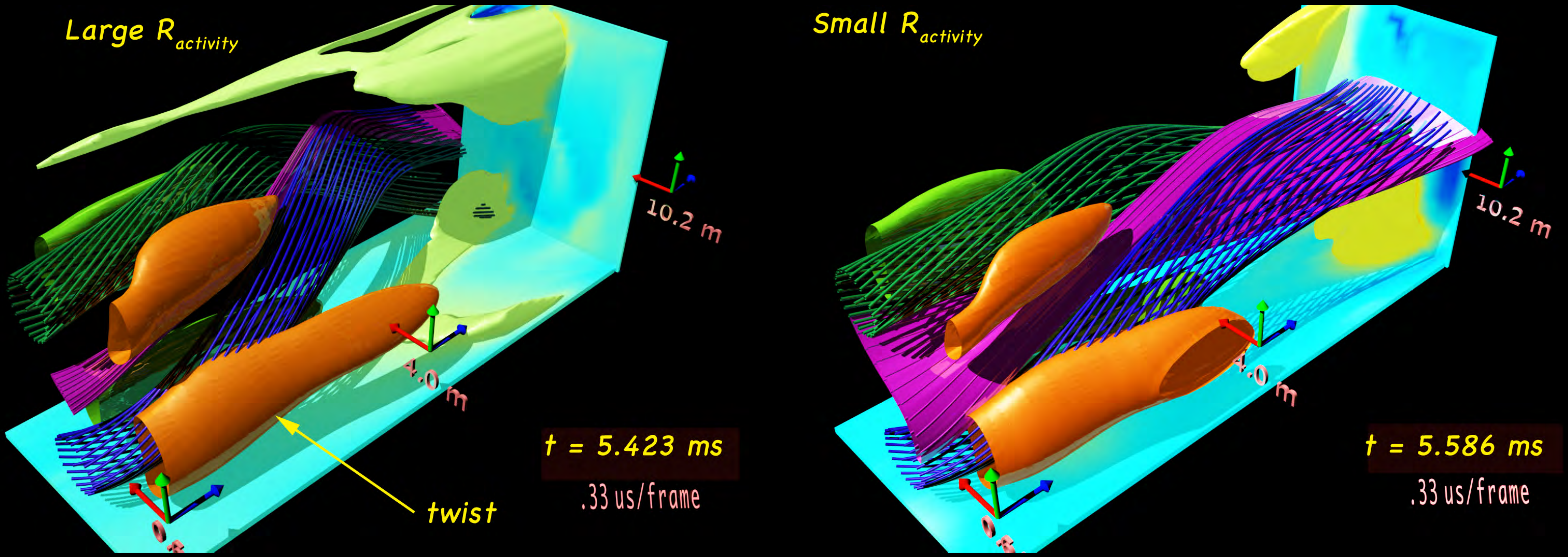
integration over all field lines





z (m)
8
6
4
2
t (ms)
5.0
5.5
6.0
Reconnection activity (positive)
Twist (positive)

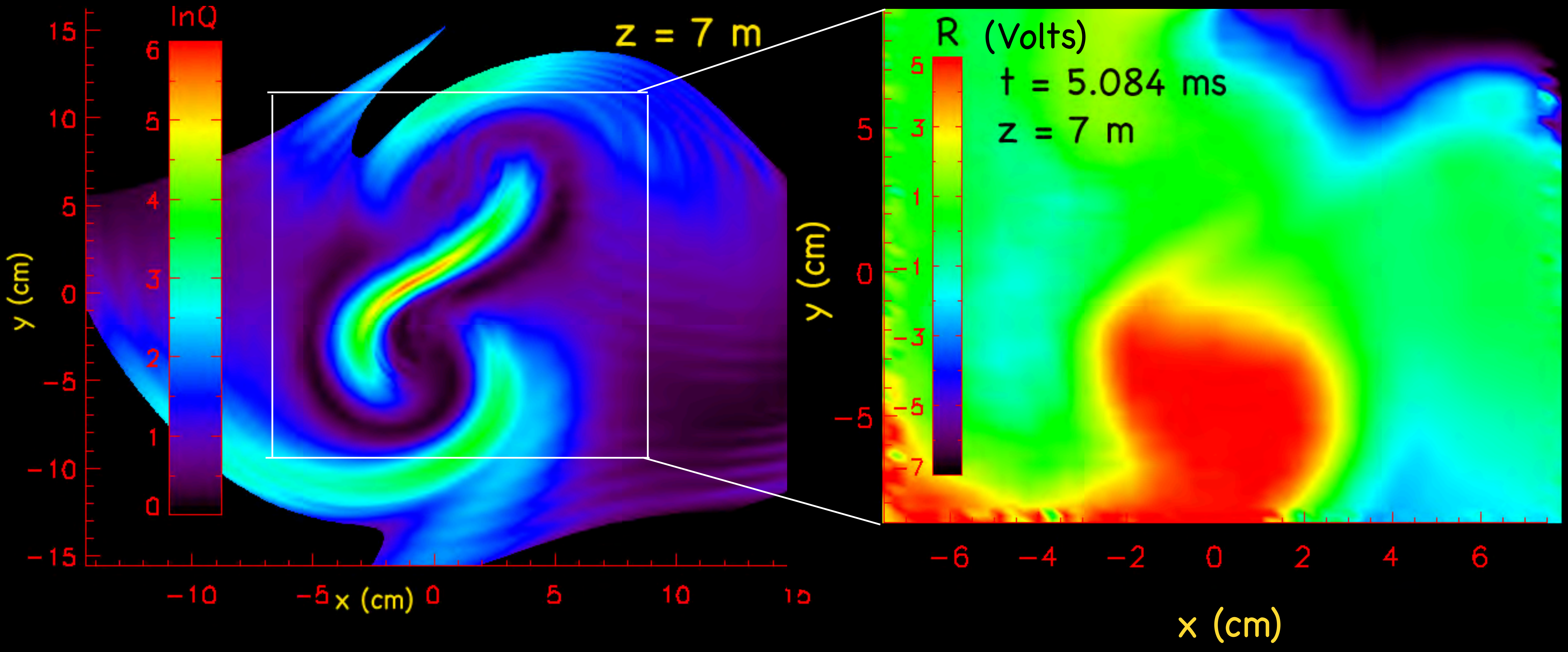

Reconnective activity , Twist
R = -8V
T = -.08
z = 4 m
R = +5V
z= 9.6 m
z = 4.7 m
z= 2.35 m
t = 5.496 ms
T = .08



The QSL and field lines are highly deformed when R is large

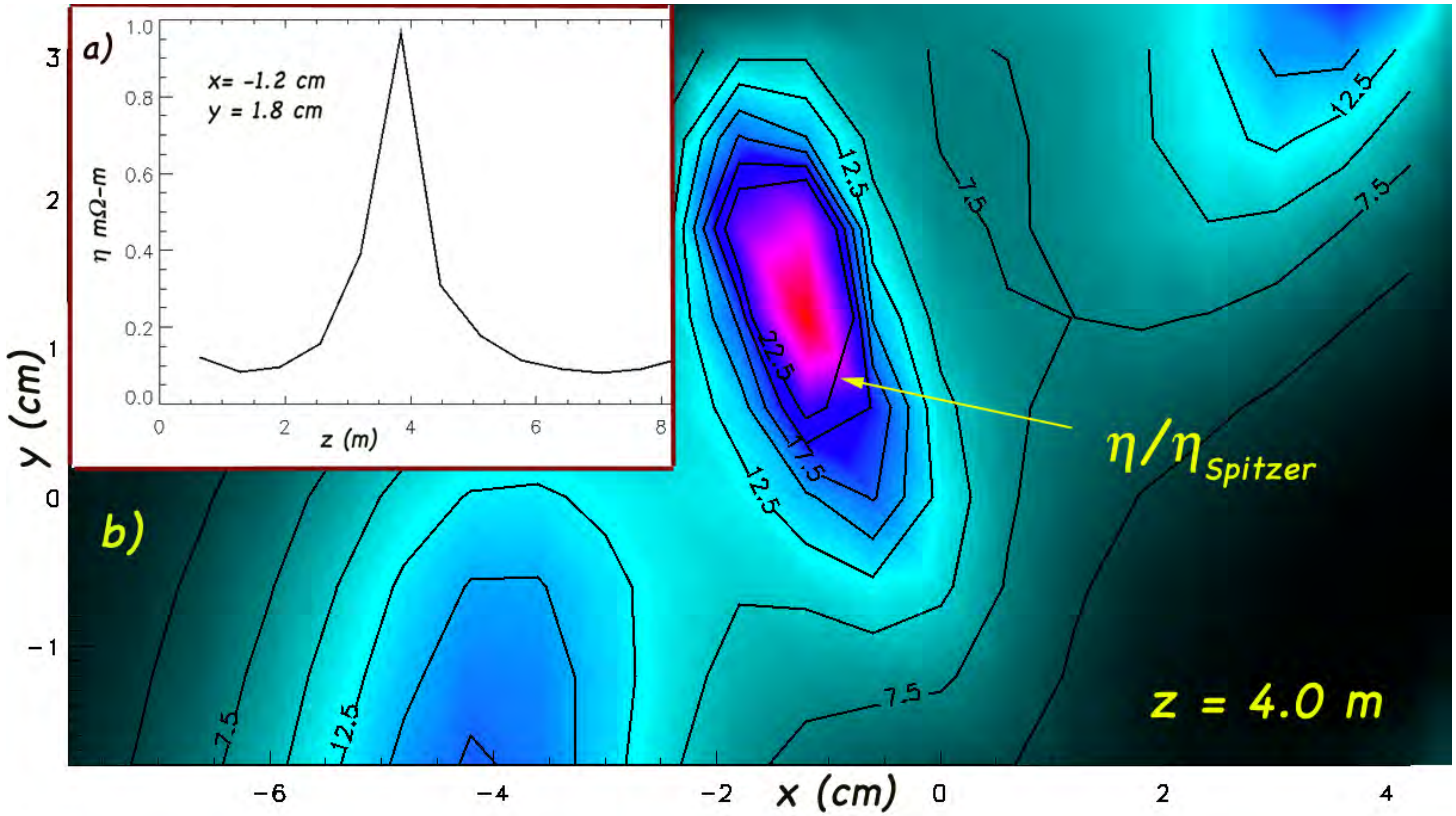
Ln (QSL)

Reconnective Activity



t = 5.084 ms

Kubo Resitivity



1) Winding number is largest on both sides of the QSL

2) Reconnective Activity (RA) is large on the edges of the ropes

3) Reconnective activity can become large at the same axial location that the Twist becomes small

4) When Reconnective Activity is large the QSL is highly deformed

5) This study has unveiled additional reconnection regions outside the largest QSL a place that researchers would not traditionally look

Two interacting flux ropes:

Twist and writhe about themselves , wrap around each other

Collide when they are kink unstable

Magnetic field line reconnection occurs at each collision

Ohms law for flux ropes is non-local

The resistivity can be deduced using the Kubo theory

Changes in flux rope helicity can also be used to derive $\langle \eta_{\|} \rangle$

Flux ropes are chaotic